When Do Power Shifts Cause War?

Reexamining Commitment Problems as a Source of Rational War

Online Appendices and Proofs

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These online appendices include proofs for various claims made in the primary paper. Appendix B shows that given stationary strategies, the model solution is unique up to indifference. Appendix C shows that A prefers to demand the strategic object whenever $\delta_A > \delta_B$ and allow B to have the strategic object whenever $\delta_A < \delta_B$. Appendix D shows that this optimal offer is always preferable to war. Appendix E proves the existence of the two mixed strategies in history-A. Finally, Appendix F discusses the solution when $\sigma = 0$, such that there is no strategic object. It shows two things: first, that given stationary strategies, there is never war when $\sigma = 0$. Second, while war may occur without stationary strategies, there is always an implementable bargain that is Pareto superior to war when $\sigma = 0$.

Before discussing the specific proofs, it is worth reviewing the model solution. As noted in the text, I largely confine my analysis to situations involving stationary strategies. In Appendix F, I extend the analysis to include possible punishment strategies when $\sigma = 0$.

Within the infinite horizon phase, the actions in history-A and history-B are interdependent; what each player will do in history-A depends on what they expect to happen in history-B and vice-versa. (What happens in period 1 can be derived from the infinite horizon phase by backward induction). We can begin to unravel this complicated solution space by assuming that the minimum offers that B will accept are possible. This creates a simply set of equations allowing the minimum offer state B will accept (1-t) to be identified, based off of the probable outcome of the war. This in turn creates a second set of equations, allowing state A's actions to be determined. I will refer to this situation, where all the minimal offers that B will accept are possible, as the basic situation and basic t-values. As will be shown, these basic t-values are always mutually preferable to war.

However, as noted in the text, there are situations where these basic t-values are not possible. It is possible that $t^{\dagger}_{hB,\sigma} < 0$. In this case, it is impossible for A to get B to accept an offer where A gets the strategic object, as A would have to offer B more than the entire value of the territory. This can lead to situations where A makes an unserious offer that they know B will reject, resulting in war. However, this has no other effect on the solution, as B's minimally acceptable offers are dictated by their expected value of war.

It is also possible for all of the basic t-values to be greater than 1. In this case, B will allow A to have the entire territory. Note that among the basic t-values, $t^{\dagger}_{hB,\sigma} \leq t^{\dagger}_{hB,\neg\sigma} \leq t^{\dagger}_{hA,\sigma} \leq t^{\dagger}_{hA,\neg\sigma}$. As noted in the text, this can lead to situations where A chooses to make an unserious offer, resulting in war.

Where a given t-value is greater than 1, A could also make offers demanding the entire territory. In this case, B would be willing to allow A to have even more, but A is already making the maximum demand possible. This situation has impacts on the other offers. Because A cannot demand as much as B would accept, B's expected value in the other stages diverges from what they would get from war. This requires the calculations of alternative t-values. These only need to be calculated for situations where A makes offers $t_{hA,\neg\sigma} = 1$ and $t_{hA,\sigma} = 1$. Because $t^{\dagger}_{hB,\sigma} \leq t^{\dagger}_{hB,\neg\sigma} \leq t^{\dagger}_{hA,\sigma} \leq t^{\dagger}_{hA,\neg\sigma}$, anytime-A would offer $t_{hB,\neg\sigma} = 1$ and $t_{hB,\sigma} = 1$ in history-B, A is already offering $t_{hA,\sigma} = 1$ in history-A. I refer to the situation where A offers $t_{hA,\neg\sigma} = 1$ as the first alternate t-values, denoted by ‡ and the situation where A offers $t_{hA,\sigma} = 1$ as the second alternate t-values, denoted by ‡†. These alternate t-values also lead to two mixed strategies in history-A, corresponding t-values in history-B (denoted by ‡‡ and ‡ ‡ †), which is discussed further in the proof for that section.

Thus, the solution can be divided into the following set of t-values. The basic situation (denoted by †) occurs when A chooses to play $t^{\dagger}_{hA,\sigma}$, $t^{\dagger}_{hA,\neg\sigma}$, or war in history-A, either because all t-values are less than 1 or A chooses $t^{\dagger}_{hA,\sigma}$ or war rather than make a suboptimal offer of $t_{hA,\neg\sigma} = 1$. The first alternative set of t-values (denoted by ‡) occurs when only $t^{\dagger}_{hA,\neg\sigma} > 1$, and A decides to play $t_{hA,\neg\sigma} = 1$ in history-A. The mixed strategy values (denoted by ‡‡ and ‡ ‡ †), discussed further below, also occur when only $t^{\dagger}_{hA,\neg\sigma} > 1$. The second alternative set of t-values (denoted by ‡†) occurs when $t^{\dagger}_{hA,\neg\sigma} \geq t^{\dagger}_{hA,\sigma} > 1$, in which case A always plays $t^{\dagger}_{hA,\sigma} = 1$. Finally, there are t-values in period 1 (denoted by ‡ ‡ ‡) for when at least one of the history-B t-values is greater than 1, and A offers $t = 1$ in both history-A and history-B.

This set of t-values structures the proofs below. For appendices C, D, and E, I simply walk through each set of t-values, and show that any deviations from the states solution are not preferred.

B Proof of solution uniqueness

When only stationary strategies are used, there is a unique solution (up to indifference) for any set of parameter values. The uniqueness of the solution with stationary strategies is shown directly from the inequalities presented in Lemmas 1-3 and the t-values and CVs presented in Appendix A.

Note first, that each of the equations in Lemmas 1-3 is expressed solely in terms of a t-value, CV and exogenous parameters. Second, each of the t-values in Appendix A.1 is expressed only in terms of exogenous parameters, while each CV in Appendix A.2 is expressed only in terms of exogenous parameters or t-values. Thus, each inequality equation (in Lemmas 1-3) can be expressed solely in terms of exogenous parameters. As these inequalities show where the actors would want to deviate from the proposed solution, the solution presented is a unique solution (up to indifference) to the game for each set of exogenous parameter values.

C Proof that among peace offers (not bounded by 0 or 1), A prefers to demand σ if $\delta_A > \delta_B$ and not otherwise

As stated in the text, where both peace offers are possible (i.e. $0 \leq t^*_\sigma \leq t^*_{\neg\sigma} \leq 1$), A prefers to demand σ when $\delta_A > \delta_B$ and allow B to have σ when $\delta_A < \delta_B$. To prove this, I will go through the various possible situations, and show that A never prefers to deviate from this strategy profile.

C.1 History A

I begin by examining th situations in history-A. First, assume that $\delta_A > \delta_B$, and so A is making the offer demanding the strategic object. A does not prefer to deviate to offering B the strategic object.

$t^\dagger_{hA,\sigma} + \delta_A CV_{A,hA,\sigma} > t^\dagger_{hA,\neg\sigma} + \delta_A CV_{A,hB,\sigma}$

After substituting CVs and t-values, this simplifies to:

$\delta_A > \delta_B$

This is the original condition, and so the deviation is not preferred.

Now assume that A prefers to offer B the strategic object($\delta_A < \delta_B$), and show that A does not prefer to deviate.

$t^\dagger_{hA,\neg\sigma} + \delta_A CV_{A,hB,\neg\sigma} > t^\dagger_{hA,\sigma} + \delta_A CV_{A,hA,\neg\sigma}$

After substituting CVs and t-values, this simplifies to:

$\delta_B > \delta_A$

Original condition, the deviation is not preferred.

In no case is the deviation preferred. Therefore, in history-A, state A always prefers to demand the strategic object when $\delta_A > \delta_B$ and allow B to have the strategic object when

$\delta_B > \delta_A$

C.2 History B

Now I look at the situation in history-B. Because the there are alternate t-values if A makes an offer of $t = 1$ in history-A, there are more situations to examine. I begin with the basic t-values, then move to the alternate t-values. First, assume that $\delta_A > \delta_B$, such that A is making the offer demanding the strategic object. I show that A does not prefer to deviate to offering B the strategic object.

$t^{\dagger}_{hB,\sigma} + \delta_A CV_{A,hA,\sigma} \geq t^{\dagger}_{hB,\neg\sigma} + \delta_A CV_{A,hB,\sigma}$

After substituting CVs and t-values, this simplifies to:

$\delta_A \geq \delta_B$

Original condition. The deviation is not preferred.

Now assume that A prefers to offer B the strategic object($\delta_A < \delta_B$), and show that A does not prefer to deviate.

$t^{\dagger}_{hB,\neg\sigma} + \delta_A CV_{A,hB,\neg\sigma} \geq t^{\dagger}_{hB,\sigma} + \delta_A CV_{A,hA,\neg\sigma}$

After substituting CVs and t-values, this simplifies to:

$\delta_B \geq \delta_A$

This is the original condition, and so the deviation is not preferred.

Now I will examine first set of alternative t-values (that is, those where A makes an offer of $t_{hA,\neg\sigma} = 1$ in history-A). These alternate t-values ($t^{\ddagger}_{hB,\neg\sigma}$ and $t^{\ddagger}_{hB,\sigma}$) only exist when A is not demanding σ, so only the situation where $\delta_A < \delta_B$ needs to be examined. The deviation is not preferred if:

$t^{\ddagger}_{hB,\neg\sigma} + \delta_A CV^{\ddagger}_{A,hB,\neg\sigma} > t^{\ddagger}_{hB,\sigma} + \delta_A CV_{A,hA,0\neg\sigma}$

After substituting CVs and t-values, this simplifies to:

$\delta_B > \delta_A$

This is the original condition, and the deviation is not preferred.

Now look at the second set of alternative t-values ($t^{\ddagger\dagger}_{hB,\sigma}$ and $t^{\ddagger\dagger}_{hB,\neg\sigma}$), (that is, those where A makes an offer of $t_{hA,\sigma} = 1$ in history-A).

Start with the situation where $\delta_A > \delta_B$, such that A prefers to demand σ, and show that A does not prefer to deviate to allowing B to hve σ.

$t^{\ddagger\dagger}_{hB,\sigma} + \delta_A CV_{A,hA,0\sigma} \geq t^{\ddagger\dagger}_{hB,\neg\sigma} + \delta_A CV^{\ddagger\dagger}_{A,hB,\sigma}$

After substituting CVs and t-values, this simplifies to:

$\delta_A \geq \delta_B$

This is the original condition, and the deviation is not preferred.

Now look at when $\delta_B > \delta_A$, such that A prefers to let B have σ. Note that in this case, A will stay in history A once reached no matter what, as it already gets its maximum long term expected utility in history A once $t^{\dagger}_{hA,\sigma} > 1$ A will not deviate if:

$t^{\ddagger\dagger}_{hB,\neg\sigma} + \delta_A CV^{\ddagger\dagger}_{A,hB,\neg\sigma} \geq t^{\ddagger\dagger}_{hB,\sigma} + \delta_A CV_{A,hA,0\sigma}$

After substituting CVs and t-values, this simplifies to:

$\delta_B \geq \delta_A$

This is the original condition, so the deviation is not preferred.

In no case is the deviation preferred. Therefore, in history-B, state A always prefers to demand the strategic object when $\delta_A > \delta_B$ and allow B to have the strategic object when $\delta_B > \delta_A$

C.3 Stage 1

In stage 1, both the basic t-values need to be examined, along with the alternate t-values where A makes an offer of $t = 1$ in either history-A or both history-A and history-B. I will start by examining the situations with the basic t-values, and then work through the alternate t-values.

Start by looking at the base t-values ($t^{\dagger}_{1,\sigma}$ and $t^{\dagger}_{1,\neg\sigma}$), where $\delta_A > \delta_B$ such that A prefers to demand the strategic object. The deviation is not preferred when:

$t^{\dagger}_{1,\sigma} + \delta_A CV_{A,hA,\sigma} > t^{\dagger}_{1,\neg\sigma} + \delta_A CV_{A,hB,\sigma}$

After substituting CVs and t-values, this simplifies to:

$\delta_A > \delta_B$

This is the original condition, so the deviation is not preferred.

Now look at the base t-values ($t^{\dagger}_{1,\sigma}$ and $t^{\dagger}_{1,\neg\sigma}$), where $\delta_A < \delta_B$ such that A prefers to allow B to have the strategic object. The deviation is not preferred if:

$t^{\dagger}_{1,\neg\sigma} + \delta_A CV_{A,hB,\neg\sigma} > t^{\dagger}_{1,\sigma} + \delta_A CV_{A,hA,\neg\sigma}$

After substituting CVs and t-values, this simplifies to:

$\delta_B > \delta_A$

This is the original condition, so the deviation is not preferred.

Now look at first alternate situation ($t^{\ddagger}_{1,\sigma}$ and $t^{\ddagger}_{1,\neg\sigma}$), where A offers $t_{hA,\neg\sigma} = 0$ in history A. Like for history-B, it is only necessary to examine the situation where $\delta_B > \delta_A$. The deviation is not preferred if:

$t^{\ddagger}_{1,\neg\sigma} + \delta_A CV_{A,hB,\neg\sigma} > t^{\ddagger}_{1,\sigma} + \delta_A CV_{A,hA,\neg\sigma}$

After substituting CVs and t-values, this simplifies to:

$\delta_B > \delta_A$

This is the original condition, so the deviation is not preferred.

Now look at second alternate condition($t^{\ddagger\dagger}_{1,\sigma}$ and $t^{\ddagger\dagger}_{1,\neg\sigma}$), where A offers $t_{hA,\sigma} = 0$ in history A. Start with when $\delta_A > \delta_B$, such that A prefers to demand the strategic object. The deviation is not preferred if:

$t^{\ddagger\dagger}_{1,\sigma} + \delta_A CV_{A,hA,\sigma} > t^{\ddagger\dagger}_{1,\neg\sigma} + \delta_A CV_{A,hB,\sigma}$

After substituting CVs and t-values, this simplifies to:

$\delta_A > \delta_B$

This is the original condition,so the deviation is not preferred.

Now look at when $\delta_B > \delta_A$, such that A prefers to allow B to have the strategic object.

The deviation is not preferred if:

$t^{\ddagger\dagger}_{1,\neg\sigma} + \delta_A CV_{A,hB,\neg\sigma} > t^{\ddagger\dagger}_{1,\sigma} + \delta_A CV_{A,hA,\sigma}$

After substituting CVs and t-values, this simplifies to:

$\delta_B > \delta_A$

This is the original condition, so the deviation is not preferred.

Next look at the situation where A plays a mixed strategy, mixing between war and offering $t_{hA,\neg\sigma} = 1$ in history A. Since the mixed strategy only exists if $\delta_B > \delta_A$, we only need to consider that scenario. A prefers to offer $t^{\ddagger\ddagger}_{1,\neg\sigma}$ rather than deviating to $t^{\ddagger\ddagger}_{1,\sigma}$ if:

$t^{\ddagger\ddagger}_{1,\neg\sigma} + \delta_A CV_{A,hB,\neg\sigma} > t^{\ddagger\ddagger}_{1,\sigma} + \delta_A CV_{A,hA,warmix}$

After substituting CVs and t-values, this simplifies to:

$\delta_B \frac{1-t^{\ddagger\ddagger}_{hB,\neg\sigma}-\alpha(1-\pi_2-\sigma)+\alpha c_B}{1-\alpha\delta_B(\pi_2+\sigma)} > \delta_A \frac{1-t^{\ddagger\ddagger}_{hB,\neg\sigma}-\alpha(1-\pi_2-\sigma)-\alpha c_A}{1-\alpha\delta_A(\pi_2+\sigma)}$

This is necessarily true as long as $\delta_B > \delta_A$, as required for the mixed strategy. Note that the left-hand and right-hand sides of the inequality are identical with the following exceptions. First, there is δ_B in the denominator on the left, and δ_A in the denominator on the right. As this term is negative on both sides, and $\delta_B > \delta_A$, the denominator on the left is smaller than the denominator on the right. This tends to make the overall value on the left bigger than the right. Second, there is $+\alpha c_B$ on the left and $-\alpha c_A$ on the right. As both costs must be positive, this again tends to make the left hand side bigger. Finally, the left side is multiplied by δ_B and the right by δ_A, which again would tend to make the left side bigger. Therefore every element that differs on the two sides of the inequality makes the left side bigger, and accordingly, the left side must be bigger. Therefore, the overall inequality is necessarily true, and the deviation is not preferred.

Finally look at the situation where A plays a mixed strategy, mixing between offering $t_{hA,\neg\sigma} = 1$ and offering $t^{\ddagger\ddagger\dagger}_{hA,\sigma}$ in history A. Since the mixed strategy only exists if $\delta_B > \delta_A$, we only need to consider that scenario. A prefers to offer $t^{\ddagger\ddagger\dagger}_{1,\neg\sigma}$ rather than deviating to $t^{\ddagger\ddagger\dagger}_{1,\sigma}$ if:

$t^{\ddagger\ddagger\dagger}_{1,\neg\sigma} + \delta_A CV_{A,hB,\neg\sigma} > t^{\ddagger\ddagger\dagger}_{1,\sigma} + \delta_A CV_{A,hA,peacemix}$

After substituting CVs and t-values, this simplifies to:

$\frac{\delta_B}{1-\beta\delta_B} > \frac{\delta_A}{1-\beta\delta_A}$

This must be true as long as $\delta_B > \delta_A$, which is a condition of the mixed strategy. Therefore, A does not prefer to deviate from its specified strategy.

In no case is the deviation preferred. Therefore, in stage 1, state A always prefers to demand the strategic object when $\delta_A > \delta_B$ and allow B to have the strategic object when $\delta_B > \delta_A$

C.4 final proof statement

For each situation, the deviation is not preferred. Thus, when both equalibria are possible, A prefers to demand σ when $\delta_A > \delta_B$ and allow B to have σ when $\delta_B > \delta_A$.

D Proof that optimal offer is always preferable to war

In the text, I noted that A always prefers its optimal offer (t_σ if $\delta_A > \delta_B$ and $t_\neg \sigma$ if $\delta_A < \delta_B$ to war, presuming that offer is possible ($0 < t < 1$). As these offers are defined as the minimum B would accept rather than fight, B would also prefer this offer to war.

The proof will mirror that in the previous section. I will go through each possible scenario, and show that A does not prefer to deviate to making an offer that would result in war.

D.1 History A

I will start in history-A. First I examine the situation where $\delta_A > \delta_B$, such that A offers $t^{\dagger}_{hA,\sigma}$. A will not deviate to war if:

$t^{\dagger}_{hA,\sigma} + \delta_A CV_{A,hA,\sigma} \geq (\pi_2 + \sigma)(1 + \delta_A(t^{\dagger}_{hA,\sigma} + \delta_A CV_{A,hA,\sigma})) + (1 - \pi_2 - \sigma)(\delta_A(t^{\dagger}_{hB,\sigma} + \delta_A CV_{A,hA,\sigma})) - c_A$

After substituting CVs and t-values, this simplifies to:

$\sigma(1-\pi_2-\sigma)(\delta_A-\delta_B)+(1-\sigma\delta_B)(c_A+c_B)\geq 0$

This is necessarily true. $0\leq\pi_2+\sigma\leq 1$, therefore $\sigma(1-\pi_2-\sigma)$ is necessarily positive. $\delta_A\geq\delta_B$ in this case, therefore the first phrase is necessarily positive. $0\leq\sigma\leq 1$ and $0\leq\delta_B\leq 1$, therefore $1-\sigma\delta_B$ is necessarily positive. Both c_A and c_B are positive, therefore the second phrase is necessarily positive. Accordingly the equation is greater than 0, and deviating to war is not preferred.

Now I examine the situation where $\delta_B>\delta_A$, such that A is making the offer $t^{\dagger}_{hA,\neg\sigma}$. A will not deviate to war if:

$t^{\dagger}_{hA,\neg\sigma}+\delta_A CV_{A,hB,\neg\sigma}\geq(\pi_2+\sigma)(1+\delta_A(t^{\dagger}_{hA,\neg\sigma}+\delta_A CV_{A,hB,\neg\sigma}))+(1-\pi_2-\sigma)(\delta_A(t^{\dagger}_{hB,\neg\sigma}+\delta_A CV_{A,hB,\neg\sigma}))-c_A$

After substituting CVs and t-values, this simplifies to:

$(\pi_2\sigma+\sigma^2)(\delta_B-\delta_A)+(1-\sigma\delta_B)(c_A+c_B)\geq 0$

This is necessarily true. $\delta_B\geq\delta_A$ in this case, so $\delta_B-\delta_A\geq 0$, and thus the first phrase is positive. $0\leq\sigma\leq 1$ and $0\leq\delta_B\leq 1$, so $1-\sigma\delta_B\geq 0$, thus the second phrase is positive and the whole equation is positive.

Therefore, A never prefers to deviate to war from its optimal offers, provided A can actually make those offers.

D.2 History B

Now, I will examine the situations in history-B. I will again start with the base offers, and then move to the alternate offers, where A makes an offer $t=1$ in history-A.

I start with base offer where $\delta_A>\delta_B$. A prefers to make this offer rather than deviating to war if:

$t^{\dagger}_{hB,\sigma}+\delta_A CV_{A,hA,\sigma}\geq\pi_2(1+\delta_A(t^{\dagger}_{hA,\sigma}+\delta_A CV_{A,hA,\sigma}))+(1-\pi_2)(\delta_A(t^{\dagger}_{hB,\sigma}+\delta_A CV_{A,hA,\sigma}))-c_A$

After substituting CVs and t-values, this simplifies to:

$(\sigma - \pi_2\sigma)(\delta_A - \delta_B) + (1 - \sigma\delta_B)(c_A + c_B) \geq 0$

This is necessarily true. $0 \leq \pi_2 \leq 1$ by definition, therefore $\pi_2\sigma \leq \sigma$, therefore $\sigma - \pi_2\sigma \geq 0$. $\delta_A \geq \delta_B$ in this case, therefore $\delta_A - \delta_B \geq 0$. Therefore, the first phrase is positive. $0 \leq \sigma \leq 1$ and $0 \leq \delta_B \leq 1$ by definition, therefore $\sigma\delta_B \leq 1$, therefore $1 - \sigma\delta_B \geq 0$. c_A and c_B are both positive, therefore the second phrase is also necessarily positive, making the equation necessarily true.

Now I look at the base t-value when $\delta_B > \delta_A$. A prefers to make this offer rather than deviating to war if:

$t^{\dagger}_{hB,\neg\sigma} + \delta_A CV_{A,hB,\neg\sigma} \geq \pi_2(1 + \delta_A(t^{\dagger}_{hA,\neg\sigma} + \delta_A CV_{A,hB,\neg\sigma})) + (1 - \pi_2)(\delta_A(t^{\dagger}_{hB,\neg\sigma} + \delta_A CV_{A,hB,\neg\sigma})) - c_A$

After substituting CVs and t-values, this simplifies to:

$\pi_2\sigma(\delta_B - \delta_A) + (1 - \sigma\delta_B)(c_A + c_B) \geq 0$

This is necessarily true. $\delta_B \geq \delta_A$ in this case, so the first phrase is positive. $0 \leq \sigma \leq 1$ and $0 \leq \delta_B \leq 1$ by definition, so $1 - \sigma\delta_B$ is positive, and thus the second phrase is also positive. This makes the whole equation positive.

Now look at the first alternate t-value ($t^{\ddagger}_{hB,\neg\sigma}$). Again, it is only necessary to look at the situation where $\delta_B > \delta_A$. A prefers to make this offer rather than deviating to war if:

$t^{\ddagger}_{hB,\neg\sigma} + \delta_A CV_{A,hB,\neg\sigma} \geq \pi_2(1 + \delta_A(1 + \delta_A CV^{\ddagger}_{A,hB,\neg\sigma})) + (1 - \pi_2)(0 + \delta_A CV^{\ddagger}_{A,hB,\neg\sigma}) - c_A$

After substituting CVs and t-values, this simplifies to:

$(1 + \pi_2\delta_B)(1 - \pi_2 + c_A) - (1 + \pi_2\delta_A)(1 - \pi_2 - c_B) \geq 0$

This is necessarily true. As $\delta_B > \delta_A$ in this case, $1 + \pi_2\delta_B > 1 + \pi_2\delta_A$. Also, as both costs must be positive, $1 - \pi_2 + c_A > 1 - \pi_2 - c_B$. Therefore, the first term is greater than the second term, and the whole equation must be positive. So, the deviation is not preferred.

Now I will examine the second alternate t-values ($t^{\ddagger\dagger}_{hB,\sigma}$ and $t^{\ddagger\dagger}_{hB,\neg\sigma}$). I start with the situation where $\delta_A > \delta_B$. A prefers to offer $t^{\ddagger\dagger}_{hB,\sigma}$ rather than deviating to war if:

$t^{\ddagger\dagger}_{hB,\sigma} + \delta_A CV_{A,hB,1\sigma} \geq \pi_2(1 + \delta_A CV_{A,hA,1\sigma}) + (1 - \pi_2)(0 + \delta_A(t^{\ddagger\dagger}_{hB,\sigma} + \delta_A CV_{A,hA,1\sigma})) - c_A$

After substituting CVs and t-values, this simplifies to:

$(\delta_A - \pi_2\delta_A)(1 - \pi_2) + \pi_2(\delta_B - \pi_2\delta_B) + c_B(1 - \delta_A + \pi_2\delta_A) + c_A(1 - \delta_B + \pi_2\delta_B) \geq 0$

This is necessarily true. As $\pi_2 \leq 1$, $(\delta_A - \pi_2\delta_A)$ and $(\delta_B - \pi_2\delta_B)$ are both positive, so each of the first two terms is positive. Similarly, $(1-\delta_A+\pi_2\delta_A)$ and $(1-\delta_B+\pi_2\delta_B)$ are both positive, so the last two terms are both positive. Therefore, the entire equation is positive. So, A does not prefer to deviate to war.

Now I look at the second alternate t-value where $\delta_B > \delta_A$. A prefers to offer $t^{\ddagger\dagger}_{hB,\neg\sigma}$ rather than deviating to war if:

$t^{\ddagger\dagger}_{hB,\neg\sigma} + \delta_A CV^{\ddagger\dagger}_{A,hB,\neg\sigma} \geq \pi_2(1 + \delta_A CV_{A,hA,1\sigma}) + (1 - \pi_2)(0 + \delta_A CV_{A,hB,1\neg\sigma}) - c_A$

$(1 - \delta_A - \delta_B + \delta_A\delta_B + \pi_2\delta_B - \pi_2\delta_A\delta_B)(1 - \pi_2 + c_A) - (1 - \delta_A - \delta_B + \delta_A\delta_B + \pi_2\delta_A - \pi_2\delta_A\delta_B)(1 - \pi_2 - c_B) \geq 0$

This is necessarily true. $(1 - \pi_2 + c_A) \geq (1 - \pi_2 - c_B)$, as both costs are positive. Also, $(1 - \delta_A - \delta_B + \delta_A\delta_B + \pi_2\delta_B - \pi_2\delta_A\delta_B) \geq (1 - \delta_A - \delta_B + \delta_A\delta_B + \pi_2\delta_A - \pi_2\delta_A\delta_B)$. The only difference in those terms, is that the first includes $\pi_2\delta_B$, while the second includes $\pi_2\delta_A$, of which both are positive. As, $\delta_B > \delta_A$ in this case, the first must be greater than the second. Therefore, the whole first part of the equation must be greater than the second, meaning that the entire equation is positive. Therefore, A does not prefer to deviate to war.

D.3 Stage 1

Now I will examine the situation in stage 1. Note that in each case, it is only necessary to examine the case where A makes its optimal offer in both history A and history B, as this is superior to the other offers A can make. If the equation is true for the optimal offer it will also necessarily be true for any other offer A may be forced to make.

I start with base t-value where $\delta_A > \delta_B$. A prefers to offer $t^{\dagger}_{1,\sigma}$ rather than deviating to war if:

$t^{\dagger}_{1,\sigma} + \delta_A CV_{A,hA,\sigma} \geq \pi_1(1 + \delta_A CV_{A,hA,\sigma}) + (1 - \pi_1)(\delta_A CV_{A,hB,\sigma}) - c_A$

After substituting CVs and t-values, this simplifies to:

$(c_A + c_B)(1 - \sigma\delta_B) + (\delta_A - \pi_1\delta_A)\sigma - (\delta_B - \pi_1\delta_B)\sigma \geq 0$

This is necessarily true. Because $\delta_A > \delta_B$, $(\delta_A - \pi_1\delta_A) > (\delta_B - \pi_1\delta_B)$, therefore $(\delta_A - \pi_1\delta_A)\sigma - (\delta_B - \pi_1\delta_B)\sigma$ is positive, and the entire equation is positive. So, A does not prefer to deviate to war.

Now look at base situation where $\delta_B > \delta_A$. A prefers to offer $t^{\dagger}_{1,\neg\sigma}$ rather than deviating to war if:

$t^{\dagger}_{1,\neg\sigma} + \delta_A CV_{A,hB,\neg\sigma} \geq \pi_1(1 + \delta_A CV_{A,hA,\neg\sigma}) + (1 - \pi_1)(\delta_A CV_{A,hB,\neg\sigma}) - c_A$

After substituting CVs and t-values, this simplifies to:

$(c_A + c_B)(1 - \sigma\delta_B) + \pi_1\sigma\delta_B - \pi_1\sigma\delta_A \geq 0$

This is necessarily true. As $\delta_B > \delta_A$ then $\pi_1\sigma\delta_B > \pi_1\sigma\delta_A$ and the whole equation is positive. Therefore, A does not prefer to deviate to war.

Now I examine the first alternate t-value ($t^{\ddagger}_{1,\neg\sigma}$), where A offers $t_{hA,\neg\sigma} = 1$ in history-A and $t^{\ddagger}_{hB,\neg\sigma}$ in history-B. As before, we only need to look at base situation where $\delta_B > \delta_A$, A does not prefer to deviate from offering $t^{\ddagger}_{1,\neg\sigma}$ to war if:

$t^{\ddagger}_{1,\neg\sigma} + \delta_A CV_{A,hB,\neg\sigma} \geq \pi_1(1 + \delta_A CV_{A,hA,1\neg\sigma}) + (1 - \pi_1)(\delta_A CV_{A,hB,\neg\sigma}) - c_A$

After substituting CVs and t-values, this simplifies to:

$(c_A + c_B)(1 + \pi_2\delta_B) + \pi_1\delta_B(1 - \pi_2 - c_B) - \pi_1\delta_A(1 - \pi_2 - c_B) \geq 0$

This is necessarily true. Because $\delta_B > \delta_A$, then $\pi_1\delta_B > \pi_1\delta_A$, and the entire equation is positive. (As long as $1 - \pi_2 - c_B > 0$; if that is negative, then $t^{\dagger}_{hB,\neg\sigma} > 1$, and A offers $t = 1$ in both history-A and history-B, which is a different situation)

Now I examine the situation involving the second alternate t-values ($t^{\ddagger\dagger}_{1,\sigma}$ and $t^{\ddagger\dagger}_{1,\neg\sigma}$), where A offers $t_{hA,\sigma} = 1$ in history A. I Start with situation where $\delta_A > \delta_B$. A prefers to offer $t^{\ddagger\dagger}_{1,\sigma}$ instead of deviating to war if:

$t^{\ddagger\dagger}_{1,\sigma} + \delta_A CV_{A,hA,1\sigma} \geq \pi_1(1 + \delta_A CV_{A,hA,1\sigma}) + (1 - \pi_1)(\delta_A CV_{A,hB,\sigma}) - c_A$

After substituting CVs and t-values, this simplifies to:

$(c_A + c_B)(1 - \delta_B + \pi_2\delta_B) + (\delta_A - \pi_1\delta_A)(1 - \pi_2 - c_B) - (\delta_B - \pi_1\delta_B)(1 - \pi_2 - c_B) \geq 0$

This is necessarily true as long as $1-\pi_2-c_B > 0$, (if that is negative, then $t^{\dagger}_{hB,\neg\sigma} > 1$, and A offers $t = 1$ in both history-A and history-B, which is a different situation). As $\delta_A > \delta_B$, $\delta_A - \pi_1\delta_A > \delta_B - \pi_1\delta_B$, which makes the whole equation positive. Thus, A does not prefer to deviate to war.

Now look at second alternate t-value ($t^{\ddagger\dagger}_{1,\neg\sigma}$ where $\delta_B > \delta_A$. A prefers to offer $t^{\ddagger\dagger}_{1,\neg\sigma}$ rather than deviating to war if:

$t^{\ddagger\dagger}_{1,\neg\sigma} + \delta_A CV^{\ddagger\dagger}_{A,hB,\neg\sigma} \geq \pi_1(1 + \delta_A CV_{A,hA,1\sigma}) + (1 - \pi_1)(\delta_A CV^{\ddagger\dagger}_{A,hB,\neg\sigma}) - c_A$

After substituting CVs and t-values, this simplifies to:

$(1 - \delta_A)(c_A + c_B)(1 - \delta_B + \pi_2\delta_B) + (\pi_1\delta_B - \pi_1\delta_A)(1 - \pi_2 - c_B) \geq 0$

This is necessarily true. Each term in the first part is positive. As $\delta_B > \delta_A$ in this case, the second part is also positive (as long as $1-\pi_2-c_B > 0$, if that is negative, then $t^{\dagger}_{hB,\neg\sigma} > 1$, and A offers $t = 1$ in both history-A and history-B, which is a different situation). Therefore, the entire equation is positive.

Now I will examine the situations where A is playing a mixed strategy in the infinite horizon phase. Since the mixed strategies only exist when $\delta_A < \delta_B$, we only need to examine that scenario.

I start by examining the situation where A mixes between offering $t_\sigma = 1$ and war. I show that A does not wish to deviate from offering $t^{\ddagger\ddagger}_{\neg\sigma}$ to war. Deviation is not preferred if:

$t^{\ddagger\ddagger}_{1,\neg\sigma} + \delta_A CV^{\ddagger\ddagger}_{A,hB,\neg\sigma} \geq \pi_1(1 + \delta_A CV_{A,hA,warmix}) + (1 - \pi_1)(\delta_A CV^{\ddagger\ddagger}_{A,hB,\neg\sigma}) - c_A$

This simplifies to:

$(c_A + c_B)\delta_A\pi_2(\pi_2 + \sigma) - \pi_1(1 - \pi_2 - \sigma - \delta_A\sigma + c_A + c_A\delta_A\pi_2 + c_B\delta_A\pi_2 + c_B\delta_A\sigma) \geq 0$

The first term is necessarily positive. In the second term, $1-\pi_2-\sigma-\delta_A\sigma+c_A+c_A\delta_A\pi_2+c_B\delta_A\pi_2 + c_B\delta_A\sigma \leq 0$ as long as $(1 - \pi_2 - \sigma + c_A)(1 + \pi_2\delta_B) - (\pi_2\delta_A + \sigma\delta_A)(1 - \pi_2 - c_B) \leq 0$

(which is one of the mixed strategy conditions) which turns the second term positive as well.

$1-\pi_2-\sigma-\delta_A\sigma+c_A+c_A\delta_A\pi_2+c_B\delta_A\pi_2+c_B\delta_A\sigma \leq (1-\pi_2-\sigma+c_A)(1+\pi_2\delta_B)-(\pi_2\delta_A+\sigma\delta_A)(1-\pi_2-c_B)$

$0 < (1-\pi_2-\sigma+c_A)(\pi_2\delta_B+-\pi_2\delta_A)$

As $\delta_B > \delta_A$ in this case, this is necessarily true. Therefore A does not prefer to deviate from offering $t^{\ddagger\ddagger}_{1,\neg\sigma}$ to war.

Now I look at the situation where A mixes between the two peace offers in history-A ($t^{\ddagger\ddagger\dagger}_{hA,\sigma}$ and $t_{hA,\neg\sigma}=1$. A prefers to offer $t^{\ddagger\ddagger\dagger}_{1,\neg\sigma}$ rather than deviating to war if:

$t^{\ddagger\ddagger\dagger}_{1,\neg\sigma}+\delta_A CV^{\ddagger\ddagger\dagger}_{A,hB,\neg\sigma} \geq \pi_1(1+\delta_A CV_{A,hA,peacemix})+(1-\pi_1)(\delta_A CV^{\ddagger\ddagger\dagger}_{A,hB,\neg\sigma})-c_A$

This simplifies to:

$(\pi_1\delta_B-\pi_1\delta_A)(1-\beta\delta_A)(1-t^{\ddagger\ddagger\dagger}_{hB,\neg\sigma})+(c_A+c_B)(1-\beta\delta_A)(1-\beta\delta_B) \geq 0$

This is necessarily true, as necessarily $t^{\ddagger\ddagger\dagger}_{hB,\neg\sigma} \leq 1$, $\beta \leq 1$ and $\delta_A < \delta_B$ for the mixed strategy to exist. Therefore each term is positive, and the overall equation is positive. So, A does not prefer to deviate from offering $t^{\ddagger\ddagger\dagger}_{hB,\neg\sigma}$ to war.

Finally, I look at situation where A makes an accepted offer of $t_\sigma = 1$ in both history A and history B (this occurs if $t^{\dagger}_{hB,\neg\sigma}$, $t^{\dagger}_{hA,\sigma}$, and $t^{\dagger}_{hA,\neg\sigma}$ are all greater than 1). In this case, $t^{\ddagger\ddagger\ddagger}_{1,\sigma}=t^{\ddagger\ddagger\ddagger}_{1,\neg\sigma}$, and so we can examine both cases where $\delta_A > \delta_B$ and $\delta_B > \delta_A$ together.

$t^{\ddagger\ddagger\ddagger}_{1,\neg\sigma}+\delta_A CV_{A,hB,1\sigma} \geq \pi_1(1+\delta_A CV_{A,hA,1\sigma})+(1-\pi_1)(\delta_A CV_{A,hB,1\neg\sigma})-c_A$

After substituting CVs and t-values, this simplifies to:After substituting CVs and t-values, this simplifies to:

$c_A+c_B \geq 0$

This is necessarily true, so A does not prefeer to deviate to war.

D.4 Final proof statement

In no case does A prefer war over its optimal t-value. Therefore, as long as A's preferred t-value is possible, A will prefer to make that offer rather than make an offer that would result in war.

E Proof of the existence of the mixed strategy

I will prove the existence of the two mixing strategies in two stages. First, I will show that there may exist two ranges where no pure strategy exists. Then, I will show that in this range deviations from the mixing strategy are not preferred. In all cases, I follow the single-deviation principle, where A may deviate for one round, and then returns to its on-path strategy.

E.1 Show that there are ranges where no pure strategy exists in history A - between two peace offers

First, I will compare the two peace offers - where A demands σ and where A does not. Begin by determining cutoff where A prefers not to deviate from making an on-path offer of $t^{\dagger}_{hA,\sigma}$ to an offer of $t_{hA,\neg\sigma} = 1$. The cutoff is:

$$t^{\dagger}_{hA,\sigma} + \delta_A CV^{\dagger}_{A,hA,\sigma} \geq 1 + \delta_A CV^{\dagger}_{A,hB,\neg\sigma}$$

After substituting CVs and t-values, this simplifies to:

$$\sigma - \sigma\delta_A\delta_B \geq (1 - \delta_A)(1 - \pi_2 - c_B + c_B\sigma\delta_B)$$

Now I will find cutoff for when A prefers not to deviate from making an on-path offer of $t_{hA,\neg\sigma} = 1$ to offering $t^{\ddagger}_{hA,\sigma}$. Note that because A's on-path strategy is to offer $t_{hA,\neg\sigma} = 1$, which creates the alternate t-values in history-B, this creates an alternate off-path t-value in history-A. The cutoff for when A prefers its on-path strategy of offering $t_{hA,\neg\sigma} = 1$ is:

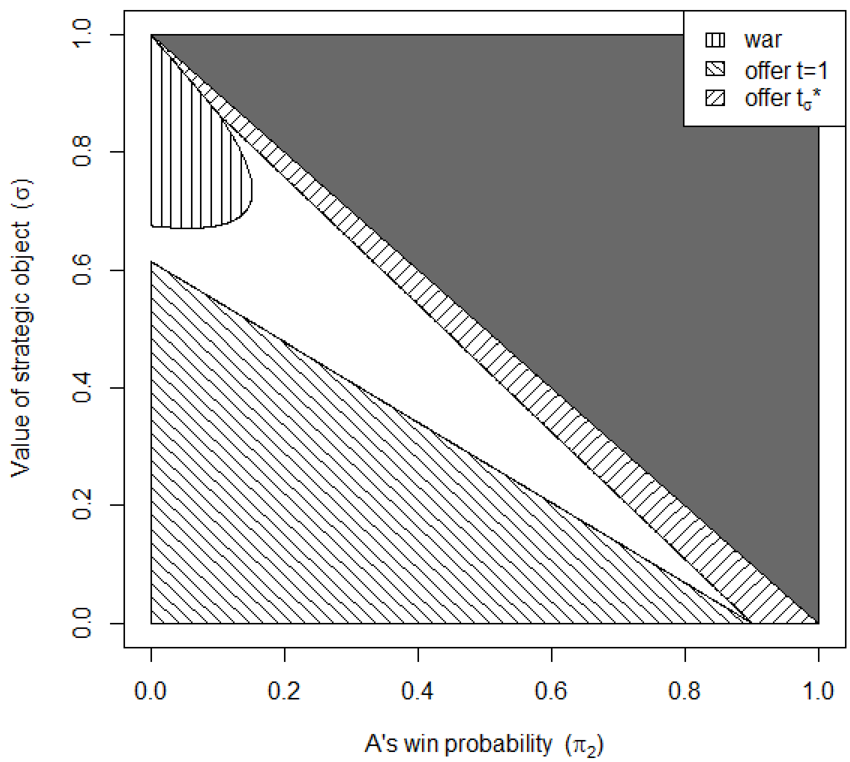
$$1 + \delta_A CV^{\ddagger}_{A,hB,\neg\sigma} \geq t^{\ddagger}_{hA,\sigma} + \delta_A(1 + \delta_A CV^{\ddagger}_{A,hB,\neg\sigma})$$

After substituting CVs and t-values, this simplifies to:

$$(1 + \delta_B - \sigma\delta_B - \delta_A)(1 - \pi_2 - c_B) - \sigma(1 + \pi_2\delta_B) \geq 0$$

Note that these two cutoffs are not identical. Thus, as can be seen in the figure E.1 below, there exist conditions where neither is true under some set of parameters. This requires a mixed strategy between the two options.

Figure E.1:
Region where no pure strategy exists in History A;
$c_A = c_B = 0.1$, $\delta_A = 0.7$, $\delta_B = 0.99$



E.2 Show that there are ranges where no pure strategy exists in history A - between war and offering $t_{hA,\neg\sigma} = 1$

I will similarly find the cutoffs between where A makes an accepted offer of $t_{hA,\neg\sigma} = 1$ and makes an unserious offer resulting in war. I start by finding the cutoff where A chooses not to deviate from an on-path strategy of offering $t_{hA,\neg\sigma} = 1$ to one round of war:

$$1 + \delta_A CV^{\ddagger}_{A,hB,\neg\sigma} \geq (\pi_2 + \sigma)(1 + \delta_A(1 + \delta_A CV^{\ddagger}_{A,hB,\neg\sigma})) + (1 - \pi_2 - \sigma)(\delta_A CV^{\ddagger}_{A,hB,\neg\sigma}) - c_A$$

After substituting CVs and t-values, this simplifies to:

$(1-\pi_2-\sigma+c_A)(1+\pi_2\delta_B)-(\pi_2\delta_A+\sigma\delta_A)(1-\pi_2-c_B)\geq 0$

Now I find cutoff for deviating from an on-path strategy of war to a single round of offering $t_{hA,\neg\sigma}=1$. While A would then stay in history B, this means that if A ever found itself back in history A, it would fight again. The cutoff where A prefers not to deviate is:

$CV_{A,hA,war}\geq 1+\delta_A CV^{\dagger}_{A,hB,\neg\sigma}$

$\frac{(1-\pi_2-\sigma)\delta_A\frac{t^{\dagger}_{hB,\neg\sigma}}{1-\delta_A}+\pi_2+\sigma-c_A}{1-\pi_2\delta_A-\sigma\delta_A}\geq 1+\delta_A\frac{t^{\dagger}_{hB,\neg\sigma}}{1-\delta_A}$

After substituting CVs and t-values, this simplifies to:

$(\pi_2\delta_A+\sigma\delta_A)(1-\pi_2-\sigma\delta_B)-(1-\sigma\delta_B)(\pi_2\delta_A+\sigma\delta_A)c_B\geq(1-\sigma\delta_B)(1-\pi_2-\sigma+c_A)$

Again note that the two equations are different. As shown in the figure E.1 above, there exists a range where neither is met.

Since there is a region where none of the pure strategies holds, there must be a region where there is a mixed strategy.

E.3 Show deviations from mixing between two peace offers are not preferred

Having shown that a mixed strategy is necessary, I will now proceed to show that the stated mixed strategy is an equilibrium. I will start by demonstrating that the stated mixed strategy between the two peace offers (mixing between $t^{\ddagger\ddagger\dagger}_{hA,\sigma}$ and $t_{hA,\neg\sigma}=1$) is preferred over possible deviations.

I start in history B, showing that A would prefer to offer $t^{\ddagger\ddagger\dagger}_{hB,\neg\sigma}$ rather than deviating to war. The deviation is not preferred if:

$t^{\ddagger\ddagger\dagger}_{hB,\neg\sigma}+\delta_A CV^{\ddagger\ddagger\dagger}_{A,hB,\neg\sigma}\geq\pi_2(1+\delta_A CV_{A,hA,peacemix})+(1-\pi_2)(\delta_A CV^{\ddagger\ddagger\dagger}_{A,hB,\neg\sigma})-c_A$

This simplifies to:

$(\pi_2\delta_B-\pi_2\delta_A)(1-\beta\delta_A)(1-t^{\ddagger\ddagger\dagger}_{hB,\neg\sigma})+(c_A+c_B)(1-\beta\delta_A)(1-\beta\delta_B)\geq 0$

This is necessarily true, as $t^{\ddagger\ddagger\dagger}_{hB,\neg\sigma}\leq 1$, $\beta\leq 1$ and $\delta_A<\delta_B$ for the mixed strategy to exist.

Therefore, A prefers the mixed strategy to war in history B.

Now, I will compare the mixed strategy offer of $t^{\ddagger\ddagger\dagger}_{hB,\sigma}$ to offering $t^{\ddagger\ddagger\dagger}_{hB,\sigma}$ in history B. The deviation is not preferred if:

$CV^{\ddagger\ddagger\dagger}_{A,hB,\neg\sigma} \geq t^{\ddagger\ddagger\dagger}_{hB,\sigma} + \delta_A CV_{A,hA,peacemix}$

After substituting CVs and t-values, this simplifies to:

$0 \geq 1 - c_B - \pi_2 - \sigma - \sigma\delta_A + c_B\sigma\delta_A$

As long as $\delta_B > \delta_A$, $t^{\dagger}_{hA,\sigma} \leq 1$ and $(1 + \delta_B - \sigma\delta_B - \delta_A)(1 - \pi_2 - c_B) - \sigma(1 + \pi_2\delta_B) < 0$ are true this must be true. Both are conditions for the mixed strategy range to occur.

$(1 + \delta_B - \sigma\delta_B - \delta_A)(1 - \pi_2 - c_B) - \sigma(1 + \pi_2\delta_B) \geq 1 - c_B - \pi_2 - \sigma - \sigma\delta_A + c_B\sigma\delta_A$

$(\delta_B - \delta_A)(1 - \sigma - \pi_2 - c_B + c_B\sigma) \geq 0$

Both of those elements are positive in this case. As $\delta_B > \delta_A$, then $\delta_B - \delta_A$ is positive. $t^{\dagger}_{hA,\sigma} \leq 1$ means that $1 - \pi_2 - \sigma - c_B + c_B\sigma\delta_B \geq 0$, which is less than the second part of the above equation. Therefore, within the conditions of the mixed strategy, the deviation is not preferred. Thus, A also prefers to play the mixed strategy offer of $t^{\ddagger\ddagger\dagger}_{hB,\sigma}$ in history-B rather than the off-path offer.

Now I will move to history A, showing that deviations from the mixed strategy are not preferred. I start with deviation from the mixed strategy to playing $t^{\ddagger\ddagger\dagger}_{hA,\sigma}$ with certainty. As, mixed strategies require the player to be indifferent between pure strategies, A should be indifferent between the mixed strategy and the deviation.

$CV_{A,hA,peacemix} \geq t^{\ddagger\ddagger\dagger}_{hA,\sigma} + \delta_A CV_{A,hA,peacemix}$

After substituting CVs and t-values, this simplifies to:

$0 \geq 0$

This is necessarily true, and A is willing to play the mixed strategy rather than play $t^{\ddagger\ddagger\dagger}_{hA,\sigma}$ with certainty.

Now compare the mixed strategy to offering $t_{hA,\neg\sigma} = 1$ with certainty. Again, A should be indifferent

$CV_{A,hA,peacemix} \geq 1 + \delta_A CV^{\ddagger\ddagger\dagger}_{A,hB,\neg\sigma}$

After substituting CVs and t-values, this simplifies to:

$0 \geq 0$

This is necessarily true. Again, A is indifferent to the deviation, and the mixed equilibrium holds.

Now, I will compare the mixed strategy to a one shot deviation to war.

$CV_{A,hA,peacemix} \geq (\pi_2 + \sigma)(1 + \delta_A CV_{A,hA,peacemix}) + (1 - \pi_2 - \sigma)(\delta_A CV^{\ddagger\ddagger\dagger}_{A,hB,\neg\sigma}) - c_A$

$(1 - \pi_2 - \sigma + c_A - \delta_A \sigma)(1 - \pi_2 - \sigma) + c_A \delta_A \pi_2 + \delta_A(\pi_2 + \sigma)(c_B - c_B \sigma) \geq 0$

This matches the cutoff between the two mixed strategies. Thus, within the conditions of the mixed strategy, A prefers mixing between $t^{\ddagger\ddagger\dagger}_{hA,\sigma}$ and $t_{hA,\neg\sigma} = 1$ to war. A thus will never deviate from the mixed strategy within the stated conditions.

It is also necessary to show that B would play the strategies of the mixed strategy. I start in history B. B should be indifferent between the mixed-strategy offer ($t^{\ddagger\ddagger\dagger}_{hB,\neg\sigma}$) and war.

$CV^{\ddagger\ddagger\dagger}_{B,hB,\neg\sigma} \geq \pi_2(\delta_B CV_{B,hA,peacemix}) + (1 - \pi_2)(1 + \delta_B CV^{\ddagger\ddagger\dagger}_{B,hB,\neg\sigma}) - c_B$

After substituting CVs and t-values, this simplifies to:

$0 \geq 0$

B is indifferent to the deviation.

Now check that B does not prefer war over $t^{\ddagger\ddagger\dagger}_{hA,\sigma}$ in history A. Again, B should be indifferent.

$1 - t^{\ddagger\ddagger\dagger}_{hA,\sigma} + \delta_B CV_{B,hA,peacemix} \geq (\pi_2 + \sigma)\delta_B CV_{B,hA,peacemix} + (1 - \pi_2 - \sigma)(1 + \delta_B CV^{\ddagger\ddagger\dagger}_{B,hB,\neg\sigma}) - c_B$

After substituting CVs and t-values, this simplifies to:

$0 \geq 0$

B is indifferent between accepting the offer and war

Finally, show B does not prefer war to accepting $t_{hA,\neg\sigma} = 1$

$0 + \delta_B CV^{\ddagger\ddagger\dagger}_{B,hB,\neg\sigma} \geq (\pi_2 + \sigma)\delta_B CV_{B,hA,peacemix} + (1 - \pi_2 - \sigma)(1 + \delta_B CV^{\ddagger\ddagger\dagger}_{B,hB,\neg\sigma}) - c_B$

After substituting CVs and t-values, this simplifies to:

$0 > 1 - \pi_2 - \sigma - c_B - \delta_A\sigma + c_B\delta_A\sigma$

This is the same final inequality as the second deviation check, (determining whether A would not deviate from its on-path strategy of $t^{\ddagger\ddagger\dagger}_{hB,\neg\sigma}$ to offering $t^{\ddagger\ddagger\dagger}_{hB,\sigma}$ in history B.) Since that was always true where the mixed strategy exists, this is always true too. B never prefers to deviate from the stated strategies in the mixed strategy.

Neither state A nor state B prefers to deviate from the mixed strategy within the stated boundaries. Therefore, the stated mixed-strategy where A mixes between $t^{\ddagger\ddagger\dagger}_{hA,\sigma}$ and $t_{hA,\neg\sigma} = 1$ in history-A is confirmed.

E.4 Show deviations from mixing between war and peace are not preferred

Now I will show that the stated mixed strategy where A mixes between offering $t_{hA,\neg\sigma} = 1$ and war in history-A exists.

I start in history B, showing that A will not deviate from offering $t^{\ddagger\ddagger}_{hB,\neg\sigma}$ to war.

$CV^{\ddagger\ddagger}_{A,hB,\neg\sigma} \geq \pi_2(1 + \delta_A CV_{A,hA,warmix}) + (1 - \pi_2)\delta_A CV^{\ddagger\ddagger}_{A,hB,\neg\sigma} - c_A$

After substituting CVs and t-values, this simplifies to:

$0 \geq 1 - \pi_2 - \sigma - \delta_A\sigma + c_A - \delta_A\sigma c_A$

This is true as long as $(1-\pi_2-\sigma+c_A)(1+\pi_2\delta_B)-(\pi_2\delta_A+\sigma\delta_A)(1-\pi_2-c_B) < 0$, which is one of the conditions for the mixed strategy. As shown below $1-\pi_2-\sigma-\delta_A\sigma+c_A-\delta_A\sigma c_A \leq (1 - \pi_2 - \sigma + c_A)(1 + \pi_2\delta_B) - (\pi_2\delta_A + \sigma\delta_A)(1 - \pi_2 - c_B)$, and so is less than 0.

$1 - \pi_2 - \sigma - \delta_A\sigma + c_A - \delta_A\sigma c_A \leq (1 - \pi_2 - \sigma + c_A)(1 + \pi_2\delta_B) - (\pi_2\delta_A + \sigma\delta_A)(1 - \pi_2 - c_B)$

$0 \leq \pi_2(\delta_B - \delta_A)(1 - \pi_2 - \sigma) + \pi_2\delta_B c_A + \pi_2\delta_A c_B + \sigma\delta_A c_B + \sigma\delta_A c_A$

This is necessarily true. $\delta_B \geq \delta_A$ in this case, and $1 - \pi_2 - \sigma \geq 0$. All the other terms are necessarily positive, and so the whole equation is positive. Therefore, A prefers to make the stated offer of $t^{\ddagger\ddagger}_{hB,\neg\sigma}$ rather than war.

Now show that A will not deviate from offering $t^{\ddagger\ddagger}_{hB,\neg\sigma}$ to offering $t^{\ddagger\ddagger}_{hB,\sigma}$ in history B.

$t^{\ddagger\ddagger}_{hB,\neg\sigma} + \delta_A CV^{\ddagger\ddagger}_{A,hB,\neg\sigma} \geq t^{\ddagger\ddagger}_{hB,\sigma} + \delta_A CV_{A,hA,warmix}$

After substituting CVs and t-values, this simplifies to:

$0 \geq 1 - \pi_2 - \sigma + c_A - \delta_A\sigma + c_A\delta_A\pi_2 + c_B\delta_A\pi_2 + c_B\delta_A\sigma$

This is necessarily true as long as $(1-\pi_2-\sigma+c_A)(1+\pi_2\delta_B)-(\pi_2\delta_A+\sigma\delta_A)(1-\pi_2-c_B) < 0$, which is one of the conditions of the mixed strategy. $(1 - \pi_2 - \sigma + c_A)(1 + \pi_2\delta_B) - (\pi_2\delta_A + \sigma\delta_A)(1 - \pi_2 - c_B)$ is greater than $\geq 1 - \pi_2 - \sigma + c_A - \delta_A\sigma + c_A\delta_A\pi_2 + c_B\delta_A\pi_2 + c_B\delta_A\sigma$, so the later must be negative.

$(1 - \pi_2 - \sigma + c_A)(1 + \pi_2\delta_B) + (-\pi_2\delta_A - \sigma\delta_A)(1 - \pi_2 - c_B) \geq 1 - \pi_2 - \sigma + c_A - \delta_A\sigma + c_A\delta_A\pi_2 + c_B\delta_A\pi_2 + c_B\delta_A\sigma$

$\pi_2 c_A(\delta_B - \delta_A) + \pi_2(\delta_B - \delta_A)(1 - \pi_2 - \sigma) \geq 0$

This is necessarily true, as $\delta_B > \delta_A$ and $1 - \pi_2 - \sigma > 0$, which must be true in the stated boundaries of the mixed strategy. Therefore, A does not prefer to deviate from the stated strategy of offering $t^{\ddagger\ddagger}_{hB,\neg\sigma}$ to offering $t^{\ddagger\ddagger}_{hB,\sigma}$ in history B.

Now I look at A's choices in history A

Now in history A, show that A will not deviate from stated strategy mixing between $t_{hA,\neg\sigma} = 1$ and war to choosing war with certainty. Like with the previous mixed strategy A should be indifferent.

$CV_{A,hA,warmix} \geq (\pi_2 + \sigma)(1 + \delta_A CV_{A,hA,warmix}) + (1 - \pi_2 - \sigma)\delta_A CV^{\ddagger\ddagger}_{A,hB,\neg\sigma} - c_A$

After substituting CVs and t-values, this simplifies to:

$0 \geq 0$

A is indifferent between the two options. Therefore, A is willing to play the mixed strategy rather than choose war with ccertainty.

Show that A will not deviate from the mixed strategy to offering $t_{hA,\neg\sigma} = 1$ with certainty. Again A should be indifferent.

$CV_{A,hA,warmix} \geq 1 + \delta_A CV_{A,hB,\neg\sigma}$

After substituting CVs and t-values, this simplifies to:

$0 \geq 0$

A is indifferent between the two options. So, A does not prefer to deviate from the mixed strategy to offering $t_{hA,\neg\sigma} = 1$ with certainty.

Finally, show that A will not deviate from the mixed strategy to offering $t^{\ddagger\ddagger}_{hA,\sigma}$.

$CV_{A,hA,warmix} \geq t^{\ddagger\ddagger}_{hA,\sigma} + \delta_A CV_{A,hA,warmix}$

After substituting CVs and t-values, this simplifies to:

$-\pi_2^2 + c_A(-1 + \pi_2 - \delta_A \pi_2 + \sigma) + \pi_2(2 + c_B \delta_A(-1 + \sigma) - (2 + \delta_A)\sigma) + (-1 + \sigma)(1 + (-1 + (-1 + c_B)\delta_A)\sigma) > 0$

This is the stated cutoff between the two mixed strategies. So, within the stated boundaries of the mixed strategy, A prefers the mixed strategy to offering $t^{\ddagger\ddagger}_{hA,\sigma}$. According, within the boundaries of the mixed strategy, A never prefers to deviate from the strategy offering $t_{hA,\neg\sigma} = 1$ and war.

Again, it is also necessary to show that B would prefer not to deviate from their stated strategies in the mixed strategy.

I start in history B, showing that B prefers not to deviate from accepting $t^{\ddagger\ddagger}_{hB,\neg\sigma}$ to war. B should be indifferent

$CV^{\ddagger\ddagger}_{B,hB,\neg\sigma} \geq \pi_2 \delta_B CV_{B,hA,warmix} + (1 - \pi_2)(1 + \delta_B CV_{B,hB,\neg\sigma}) - c_B$

After substituting CVs and t-values, this simplifies to:

$0 \geq 0$

B is indifferent between accepting the offer and war. Therefore, B will play the strategies in the mixed strategy.

Now show that B does not want to choose war if offered $t_{hA,\neg\sigma} = 1$ in history A

$0 + \delta_B CV^{\ddagger\ddagger}_{B,hB,\neg\sigma} \geq (\pi_2 + \sigma)\delta_B CV_{B,hA,warmix} + (1 - \pi_2 - \sigma)(1 + \delta_B CV^{\ddagger\ddagger}_{B,hB,\neg\sigma}) - c_B$

After substituting CVs and t-values, this simplifies to:

$1 + c_A - \pi_2 - \sigma - \delta_A\sigma + c_B\delta_A\sigma \leq 0$

This is necessarily true as long as $(1-\pi_2-\sigma+c_A)(1+\pi_2\delta_B)-(\pi_2\delta_A+\sigma\delta_A)(1-\pi_2-c_B) < 0$, which is a condition for the mixed equilibrium. $1 + c_A - \pi_2 - \sigma - \delta_A\sigma + c_B\delta_A\sigma$ is less than $(1 - \pi_2 - \sigma + c_A)(1 + \pi_2\delta_B) - (\pi_2\delta_A + \sigma\delta_A)(1 - \pi_2 - c_B)$, which is less than zero.

$1 + c_A - \pi_2 - \sigma - \delta_A\sigma + c_B\delta_A\sigma \leq (1 - \pi_2 - \sigma + c_A)(1 + \pi_2\delta_B) - (\pi_2\delta_A + \sigma\delta_A)(1 - \pi_2 - c_B)$

$0 \leq \pi_2(\delta_B - \delta_A)(1 - \pi_2 - \sigma) + \pi_2\delta_B c_A + \pi_2\delta_A c_B$

This is necessarily true. $\delta_B > \delta_A$ in this case, and $1 - \pi_2 - \sigma \geq 0$. All other terms are necessarily positive, so the whole equation is positive. Therefore B does not prefer to deviate from accepting $t_{hA,\neg\sigma} = 1$ in history A to war.

Accordingly, within the boundaries of the mixed strategy, neither state A nor state B would prefer to deviate, and the stated mixed strategy exists.

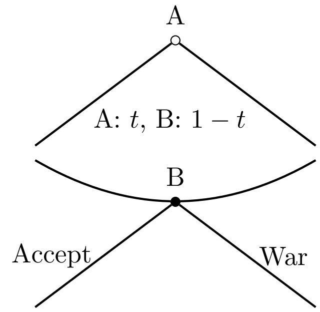
F Discussion of equilibria when $\sigma = 0$

The central finding in this paper is that when $\sigma = 0$, such that there is no strategic object, war should not occur in an conflict with perfect information. Here, I will first show that given stationary strategies, there is never an equilibrium where war occurs when $\sigma = 0$.

I then consider the equilibria that use punishment strategies. There are equilibria where war occurs. However, I will prove that these equilibria are always Pareto inferior to at least one peace equilibrium. So, we can presume that the adversaries would agree to the peace equilibrium rather than fight.

As a starting point for discussion, note that when $\sigma = 0$, the game is essentially that displayed in Figure F-1. When $\sigma = 0$, $\pi_2 + \sigma = \pi_2$, and thus history A is the same as history B, and the choice of who has the meaningless strategic object is irrelevant and can be dropped. This model would retain the distinction between the infinite horizon phase, where A's probability of war victory is π_2, and the initial stage, where A's probability of victory is π_1

Figure F.1: Simplified stage game when $\sigma = 0$



F.1 Proof that with stationary strategies, there is no war.

If stationary strategies are specified, war never occurs in this model. Within the infinite horizon phase, B will always accept any offer equal or greater than the expected value of war. Without punishment strategies, B would always be willing to divert to from any more stringent reservation value to a reservation value equal to that of the expected value of war. This means that B will always accept the following t-value.

$$1 - t^*_{IH} + \delta_B CV_{B,IH} = \pi_2(0 + \delta_B CV_{B,IH}) + (1 - \pi_2)(1 + \delta_B CV_{B,IH}) - c_B$$

$$t_{IH} = \pi_2 + c_B$$

Next, I confirm that A always prefers to make this offer this rather than deviate to making an unserious offer that would lead to war.

$$t^*_{IH} + \delta_A CV_{A,IH} \geq \pi_2(1 + \delta_A CV_{A,IH}) + (1 - \pi_2)(0 + \delta_A CV_{A,IH}) - c_A$$

$$c_A + c_B \geq 0$$

This is necessarily true. A is always willing to make an offer that B also prefers to war

Now show that there is also always a negotiated offer that both states prefer in stage 1. Again, B will accept any offer equal or greater than its expected value for fighting a war.

$$1 - t^*_1 + \delta_B CV_{B,IH} = \pi_1(0 + \delta_B CV_{B,IH}) + (1 - \pi_1)(1 + \delta_B CV_{B,IH}) - c_B$$

$$t_1 = \pi_1 + c_B$$

Now show that A again prefers to make this offer rather than make an unserious offer leading to war.

$t_1 + \delta_A CV_{A,IH} \geq \pi_1(1 + \delta_A CV_{A,IH}) + (1 - \pi_1)(0 + \delta_A CV_{A,IH}) - c_A$

$c_A + c_B \geq 0$

This is necessarily true. A is always willing to make an offer that B also prefers to war. Thus, both sides always prefer a negotiated settlement to war.

F.2 Proof that without stationary strategies, there is always a Pareto superior outcome to war.

While most of the analysis has assumed stationary strategies, the core claim that war likely never occurs if $\sigma = 0$ holds even if punishment strategies are allowed. While there may be equilibria where war occurs, these are always Pareto inferior to a peace equilibirum. My discussion of the equilibria with punishment strategies will proceed in three phases. First, I will show that divisions of the territory other than that equivalent to B's expected value of war can be maintained. Second, I will show that the existence of these divisions can enable war to occur. A punishment strategy of either sides' least favorable division can incentivize them to fight. Finally, I will show that these war equilibria are Pareto inferior to peace equilibria that exist, and the strategies required for the war equilibria are in fact substantively odd.

In the infinite horizon phase, any division of the territory that both sides prefer to war can be sustained in equilibrium, as long as $\delta_B \geq \frac{1}{2}$. There is an SPNE such that B demands some value t^*_{IH} where $\pi_2 - c_A \leq t^*_{IH} \leq \pi_2 + c_B$, similar to the equilibria in Slantchev (2003). In other words, B may be able to obtain a greater share of the territory than its value for war.

Even without punishment strategies, A is willing to make an offer meeting B's reservation value (which may be greater than B's value for war) as long as that demand is greater than A's expected value for war, which means that on A's side any offer $t^*_{IH} \geq \pi_2 - c_A$ is sustainable. A would have no reason to deviate to demanding anything less than B's reservation value.

$t^*_{IH} + \delta_A CV_{A,IH} \geq \pi_2(1 + \delta_A CV_{A,IH}) + (1 - \pi_2)\delta_A CV_{A,IH} - c_A$

$t^*_{IH} \geq \pi_2 - c_A$

B will obviously not accept any value less than its expected value for war, which requires that $t^*_{IH} \leq \pi_2 + c_B$

$1 - t^*_{IH} + \delta_B CV_{B,IH} \geq \pi_2 \delta_B CV_{B,IH} + (1 - \pi_2)(1 + \delta_B CV_{B,IH}) - c_B$

$t^*_{IH} \leq \pi_2 + c_B$

At the same time, B can sustain a reservation value greater than its value for war through a strategy where if B every deviates to a lesser reservation value, B will themselves reset their reservation value equal to that of its value to war, and that A will make that offer. To sustain an equilibrium greater than B's war value, B has to be willing to fight if its reservation value is not met. So assume A takes a greater value $t^*_{hB} + \epsilon$, B will fight if:

$\pi_2 \delta_B CV_{B,IH} + (1 - \pi_2)(1 + \delta_B CV_{B,IH}) - c_B \geq 1 - t^*_{IH} + \epsilon + \delta_B CV_{B,IH,extremal}$

$1 - \pi_2 + \delta_B CV_{B,IH} - c_B \geq 1 - t^*_{IH} + \epsilon + \delta_B CV_{B,IH,extremal}$

$1 - \pi_2 + \delta_B \frac{1 - t^*_{IH}}{1 - \delta_B} - c_B \geq 1 - t^*_{IH} + \epsilon + \delta_B \frac{1 - \pi_2 - c_B}{1 - \delta_B}$

$(t^*_{IH} - \pi_2 - c_B)(1 - \delta_B) + \delta_B(1 - t^*_{IH}) - \delta_B(1 - \pi_2 - c_B) \geq \epsilon$

$(t^*_{IH} - \pi_2 - c_B)(1 - \delta_B) - \delta_B(t^*_{IH} - \pi_2 - c_B) \geq \epsilon$

$(\pi_2 + c_B - t^*_{IH})(1 - 2\delta_B) \leq \epsilon$

$1 - 2\delta_B \leq \frac{\epsilon}{\pi_2 + c_B - t^*_{IH}}$

$\frac{1}{2} - \frac{\epsilon}{2(\pi_2 + c_B - t^*_{IH})} \leq \delta_B$

ϵ can be arbitrarily small, but is assumed to be positive. Thus, this holds true as long as $\frac{1}{2} \leq \delta_B \leq 1$. It has already been established that $t^*_{IH} \leq \pi_2 + c_B$. Thus, in the infinite horizon phase, any equilibrium can be sustained that is greater than both sides payoff for war, as long as B's shadow of the future is sufficiently long.

A similar division can be sustained in stage 1.

A is willing to make an offer meeting B's reservation value, as long as that offer is preferable to war.

$t^*_1 + \delta_A CV_{A,IH} \geq \pi_1(1 + \delta_A CV_{A,IH}) + (1 - \pi_1)\delta_A CV_{A,IH} - c_A$

$t_1^* \geq \pi_1 - c_A$

For a peace equilibrium to exist, B must be willing to both fight if its reservation value is not met, and not fight if its reservation value is met.

First, check when B would be willing to accept the reservation value if offered rather than divert to war. It is possible that reservation values less than B's value for war can be sustained if the on path infinite horizon equilibria is greater than B's value for war, by threatening to change this to the value of war.

$1 - t_1^* + \delta_B CV_{B,IH} \geq \pi_1 \delta_B CV_{B,IH,extremal} + (1 - \pi_1)(1 + \delta_B CV_{B,IH,extremal}) - c_B$

$1 - t_1^* + \delta_B CV_{B,IH} \geq 1 - \pi_1 - c_B + \delta_B CV_{B,IH,extremal}$

$t_1^* - \delta_B CV_{B,IH} \leq \pi_1 + c_B - \delta_B CV_{B,IH,extremal}$

$t_1^* - \delta_B \frac{1-t_{IH}^*}{1-\delta_B} \leq \pi_1 + c_B - \delta_B \frac{1-\pi_2-c_B}{1-\delta_B}$

$t_1^* \leq \pi_1 + c_B + \delta_B \frac{\pi_2+c_B-t_{IH}^*}{1-\delta_B}$

B can also sustain a greater reservation value than war by threatening to divert to a reservation value equal to war in the infinite horizon phase. Again, B must be willing to fight if its greater reservation value is not met.

$\pi_1 \delta_B CV_{B,IH} + (1 - \pi_1)(1 + \delta_B CV_{B,IH}) - c_B \geq 1 - t_1^* + \epsilon + \delta_B CV_{B,IH,extremal}$

$1 - \pi_1 - c_B + \delta_B CV_{B,IH} \geq 1 - t_1^* + \epsilon + \delta_B CV_{B,IH,extremal}$

$1 - \pi_1 - c_B + \delta_B \frac{1-t_{IH}^*}{1-\delta_B} \geq 1 - t_1^* + \epsilon + \delta_B \frac{1-\pi_2-c_B}{1-\delta_B}$

$t_1^* - \pi_1 - c_B + \delta_B \frac{\pi_2+c_B-t_{IH}^*}{1-\delta_B} \geq \epsilon$

ϵ can be arbitrarily small, and must be positive so it is possible to replace with 0.

$t_1^* - \pi_1 - c_B + \delta_B \frac{\pi_2+c_B-t_{IH}^*}{1-\delta_B} \geq 0$

$(\pi_1 + c_B - t_1^*)(1 - \delta_B) - \delta_B(\pi_2 + c_B - t_{IH}^*) \leq 0$

$\pi_1 + c_B - t_1^* \leq \delta_B(\pi_1 + \pi_2 + 2c_B - t_1^* - t_{IH}^*)$

$\delta_B \geq \frac{\pi_1+c_B-t_1^*}{\pi_1+\pi_2+2c_B-t_1^*-t_{IH}^*}$

Because equilibria exist where B receives a greater expected utility than war, war can also be sustained for a number of rounds. For instance, there is an equilibrium where B rejects all offers in the initial stage and for n periods of the infinite horizon phase and then changes to accepting an offer t^*_{IH} under the above equilibrium. This equilibrium is maintained by any deviation changing B's reservation value to that of its expected value of war from then on. I assume that A always offers t^*_1 and t^*_{IH}.

Starting in the first period of the infinite horizon phase, this equilibrium can be maintained as long as the following inequality is true for every value of n up to n. Note that I assume that B deviates to accepting the offer in the first stage of the infinite horizon phase, as we can deduce deviations in later rounds from this inequality.

$\frac{EV_{war}}{1-\delta_B} - \delta_B^n \frac{EV_{war}}{1-\delta_B} + \delta_B^n \frac{1-t^*_{IH}}{1-\delta_B} \geq 1 - t^*_{IH} + \delta_B \frac{1-t_{IH,extremal}}{1-\delta_B}$

$\frac{1-\pi_2-c_B}{1-\delta_B} - \delta_B^n \frac{1-\pi_2-c_B}{1-\delta_B} + \delta_B^n \frac{1-t^*_{IH}}{1-\delta_B} \geq 1 - t^*_{IH} + \delta_B \frac{1-\pi_2-c_B}{1-\delta_B}$

$0 \geq (1 - \delta_B - \delta_B^n)(t^*_{IH} - (1 - \pi_2 - c_B))$

Note that $t^*_{IH} \geq 1 - \pi_2 - c_B$, and so the second term is positive. Thus:

$\delta_B + \delta_B^n \geq 1$

As $\delta_B < 1$, this is most likely when $n = 1$. As it is necessary for the inequality to hold every value of n up to the number of rounds of war, an equilibrium exists where B rejects any offer and fights for an indefinite number of rounds as long as $\delta_B \geq \frac{1}{2}$.

Because of the above finding that the bounding condition is when n=1 in the infinite horizon phase, we can look at just two scenarios in stage 1 - where B fights no additional rounds and where B fights one round in the infinite horizon phase, and conclude that these generalize to all additional scenarios. Start with where B fights no rounds in the infinite horizon phase:

$1 - \pi_1 - c_B + \delta_B \frac{1-t^*_{IH}}{1-\delta_B} \geq 1 - t^*_1 + \delta_B \frac{1-\pi_2-c_B}{1-\delta_B}$

$\delta_B(1 - t^*_1 - (1 - \pi_1 - c_B) + 1 - t^*_{IH} - (1 - \pi_1 - c_B) - (1 - \pi_2 - c_B)) \geq 1 - t^*_1 - (1 - \pi_1 - c_B)$

$\delta_B \geq \frac{1-t^*_1-1+\pi_1+c_B}{1-t^*_1+1-t^*_{IH}-2+\pi_1+\pi_2+2c_B}$

$\delta_B \geq \frac{\pi_1+c_B-t^*_1}{\pi_1+\pi_2+2c_B-t^*_1-t^*_{IH}}$

Now for when B fights a single round in the infinite horizon phase:

$1 - \pi_1 - c_B + \delta_B(1 - \pi_2 - c_B) + \delta_B^2 \frac{1-t^*_{IH}}{1-\delta_B} \geq 1 - t^*_1 + \delta_B \frac{1-\pi_2-c_B}{1-\delta_B}$

$\delta_B(1 - t^*_1 - (1 - \pi_1 - c_B)) + \delta_B^2(1 - t^*_{IH} - (1 - \pi_2 - c_B)) \geq 1 - t^*_1 - (1 - \pi_1 - c_B)$

Note that if the cutoff for fighting only in stage 1 is true, this must be true as well. So, the bounding condition is when B fights only in stage 1.

Also note that the cutoffs ($\delta_B \geq \frac{1}{2}$ in the infinite horizon phase and $\delta_B \geq \frac{\pi_1+c_B-t_1^*}{\pi_1+\pi_2+2c_B-t_1^*-t_{IH}^*}$ in stage 1) are the same conditions for B being able to achieve a greater proportion of the territory than their expected value foor war. Thus, whenever equilibria exist where B can achieve more than their expected value for war, there are also equilibria where war actually occurs.

Having seen that war equilibria can exist it is now possible to show that these equilibria are always Pareto inferior to at least one equilibria. For simplicity, I will focus on the offers t_1^* and t_{IH}^* used to maintain the equilibrium. However, these are not necessarily the only peace equilibria that would be Pareto superior to war.

First in the infinite horizon phase, B will find the peace equilibria where t_{IH}^* is offered and accepted preferable to n rounds of war if:

$\frac{1-t_{IH}^*}{1-\delta_B} \geq \frac{EV_{B,war}}{1-\delta_B} - \delta_B^n \frac{EV_{B,war}}{1-\delta_B} + \delta_B^n \frac{1-t_{IH}^*}{1-\delta_B}$

$\frac{1-t_{IH}^*}{1-\delta_B} \geq \frac{1-\pi_2-c_B}{1-\delta_B} - \delta_B^n \frac{1-\pi_2-c_B}{1-\delta_B} + \delta_B^n \frac{1-t_{IH}^*}{1-\delta_B}$

$(1-\delta_B^n)(1-t_{IH}^*) \geq (1-\delta_B^n)(1-\pi_2-c_B)$

$t_{IH}^* \leq \pi_2 + c_B$

This is a condition for t_{IH}^* being possible, which in turn is necessary to sustain the war equilibrium, and thus B always prefers peace to war.

Now show that A also prefers offering t_{IH}^* to war:

$\frac{t_{IH}^*}{1-\delta_A} \geq \frac{EV_{A,war}}{1-\delta_A} - \delta_A^n \frac{EV_{A,war}}{1-\delta_A} + \delta_A^n \frac{t_{IH}^*}{1-\delta_A}$

$\frac{t_{IH}^*}{1-\delta_A} \geq \frac{\pi_2-c_A}{1-\delta_A} - \delta_A^n \frac{\pi_2-c_A}{1-\delta_A} + \delta_A^n \frac{t_{IH}^*}{1-\delta_A}$

$(1-\delta_A^n)(t_{IH}^*) \geq (1-\delta_A^n)(\pi_2-c_A)$

$t_{IH}^* \geq \pi_2 - c_A$

This is also a condition for t_{IH}^* to exist, which is necessary to sustain the war equilibrium, and so A also prefers peace to war. Therefore, for every war equilibria that exists, there is a peace equilibria that is preferred by both states. Note, as the equilibrium where t_{IH}^* is offered and accepted is part of the war equilibrium, this means that the war equilibrium is not internally renegotiation proof within the infinite horizon phase.

Now show that in stage 1, there is also always an equilibirium, t_1^* that is preferable to war. Given that I just showed that there is always a Pareto superior peace equilibrium in the infinite horizon phase, it is only necessary to examine the scenario where war only occurs in stage 1. If in stage 1, peace is always preferable to war without war in the infinite horizon phase, peace will also be preferable if some war occurs in the infinite horizon phase.

First show the condition under which B prefers peace:

$1 - t_1^* + \delta_B \frac{1-t_{IH}^*}{1-\delta_B} \geq 1 - \pi_1 - c_B + \delta_B \frac{1-t_{IH}^*}{1-\delta_B}$

$1 - t_1^* \geq 1 - \pi_1 - c_B$

$t_1^* \leq \pi_1 + c_B$

Now show the condition under which A also prefers peace:

$t_1^* + \delta_A \frac{t_{IH}^*}{1-\delta_A} \geq \pi_1 - c_A + \delta_A \frac{t_{IH}^*}{1-\delta_A}$

$1 - \pi_1 + c_A \geq 1 - t_1^*$

$t_1^* \geq \pi_1 - c_A$

Thus, any offer $\pi_1 - c_A \leq t_1^* \leq \pi_1 + c_B$ is mutually preferable to war. As c_A and c_B are both positive, this range must exist. In addition as B is willing to accept any division, $t_1^* \leq \pi_1 + c_B + \delta_B \frac{\pi_2 + c_B - t_{IH}^*}{1-\delta_B}$ (shown above), this equilibrium is always sustainable.

This analysis has shown three things. First, non-stationary strategies do make offers other than B's expected value for war possible. Second, there are conditions under which war equilibria exist even when $\sigma = 0$. Finally, while these war equilibria exist, in both the infinite horizon phase and the initial stage, there is always an equilibrium that is Pareto superior to the war equilibrium (as long as $\sigma = 0$).

Given that this is a perfect information game, the existence of these mutually preferred equilibria is known to both states. Thus, it seems reasonable to presume that they would negotiate one of the peace equilibria rather than fight a war that neither wants. Note also that the within the infinite horizon phase, the war equilibrium is not renegotiation proof, which reinforces the view that these are unlikely equilibria.[1]

[1] As there is no other stage 1, internal renegotiation proofness is not applicable in evaluating the equilibria in stage 1. However, since the war and peace equilibria in stage 1 follow those in the infinite horizon game, we can conclude that again the peace equilibria is more likely.