How Does War Resolve Commitment Problems? Online Appendices

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These appendices contain additional information about the models and results. Appendix A presents the substantive interpretation of the regime change model, which is similar but somewhat more complicated than the decisive war and strategic object models. The remaining appendices detail the mathematical solutions to each model and proofs of the solutions.

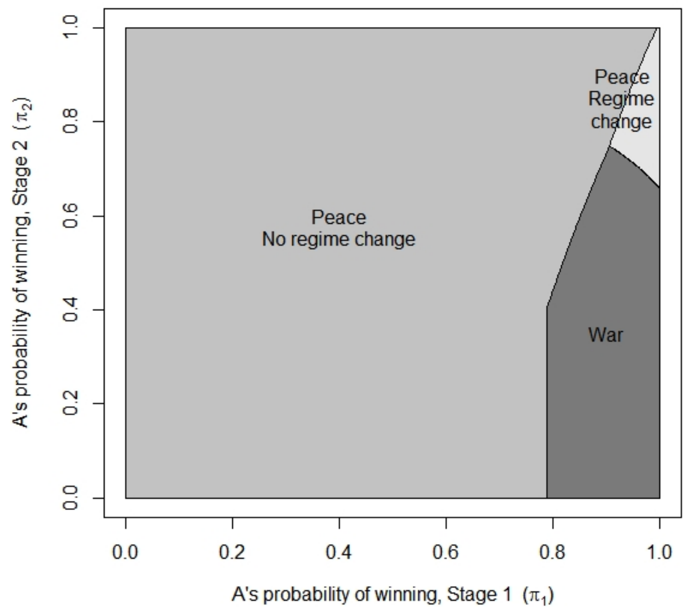
A Substantive results of the regime change and strategic object models

In the main paper, I focused on the substantive results of the decisive war model and the strategic object model, which were nearly identical. While the basic findings of the regime change model are similar, the specific elements are somewhat different. These differences arises from the asymmetry of gains and losses in the regime change model. Regime change represents a complete loss of benefits in the second stage for the initial B regime. However, A only gains benefits equal to the difference in war costs or resolve of the initial and replacement B regimes.

Figures 1 and 2 show the relationship between the probability of winning and war in the regime change model. When A has a shorter time horizon ($\delta_A < \delta_B$), the results are very similar to the other two models, as seen in figure 1. War only occurs when A has a large power advantage in the first stage. In addition, war requires some power shift between the first and second stage. However, the war range is still stretched vertically, such that the primary determinant of war is the power balance in the first stage, and smaller effect of the power balance in the second.

The logic is similar to the decisive war model and the strategic object model. When A has a shorter time horizon, they care most about the benefits in stage 1. The initial B regime is also willing to give A more of the disputed good to avoid regime change. However, when A's chances of winning in the first phase are high, A may still care enough about gaining

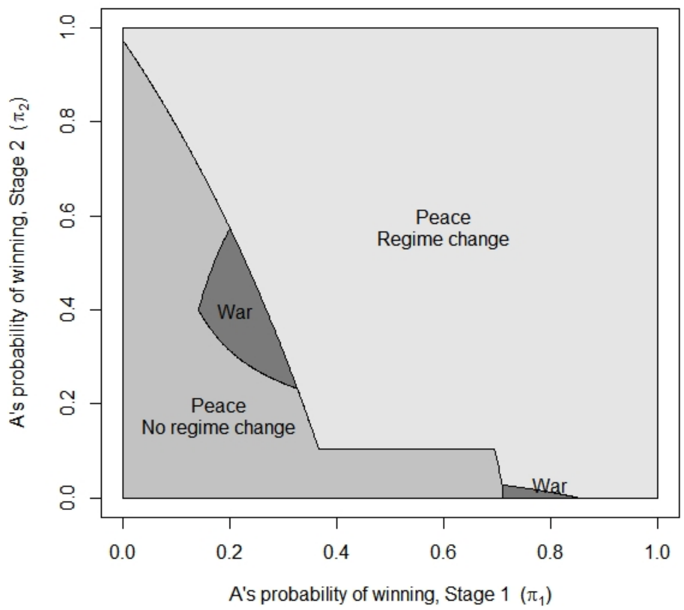
Figure 1:
A's Probability of Victory and War, $\delta_A < \delta_B$
Regime Change Model
$\rho = 0.8$, $c_A = 0.01$, $c_{Bi} = 0.01$, $c_{Br} = 0.6$, $\delta_A = 0.6$, $\delta_B = 0.9$



additional benefits in the second stage to fight. Because the initial B regime has a longer time horizon, they may also be unwilling to allow regime change without a fight. War also requires a power shift. If A also had a high chance of winning in the second stage, B would be more willing to allow regime change, as they wouldn't be giving up many second stage benefits. A would likewise feel less desire to achieve regime change, as they would already be getting substantial benefits in the second stage.

When $\delta_A > \delta_B$, the relationship between A's probability of victory and war is somewhat more complex, as shown in figure 2. However, the basic findings are similar to the decisive war model and strategic object model. First, war can still happen when $\pi_1 = \pi_2$ and there is no exogenous power shift. Also, too large an exogenous power shift generally prevents war. This occurs because of the same logic as in the other models. If A has too much power in the first stage, the initial B regime will cede control without a fight in ex-

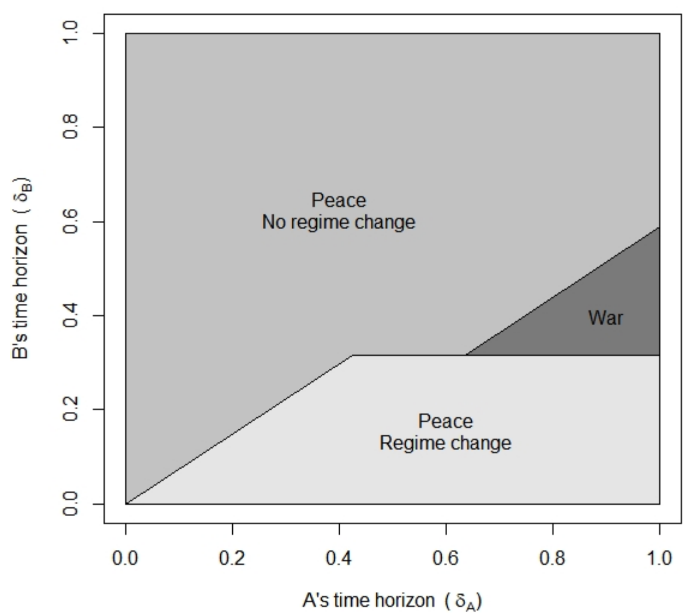
Figure 2:
A's Probability of Victory and War, $\delta_A > \delta_B$
Regime Change Model
$\rho = 0.8$, $c_A = 0.01$, $c_{Bi} = 0.01$, $c_{Br} = 0.6$, $\delta_A = 0.9$, $\delta_B = 0.6$



change for more of the disputed good in the present. The war range changes shape somewhat depending on parameter values. As the discrepancy between the actors' time horizons (δ_A and δ_{Bi}) grows, the chances of regime change if A wins (ρ) grows, or the replacement regimes war costs (c_{Br}) grow, the main war range becomes more similar to that in the decisive war model, and the remnant war range in the lower right disappears.

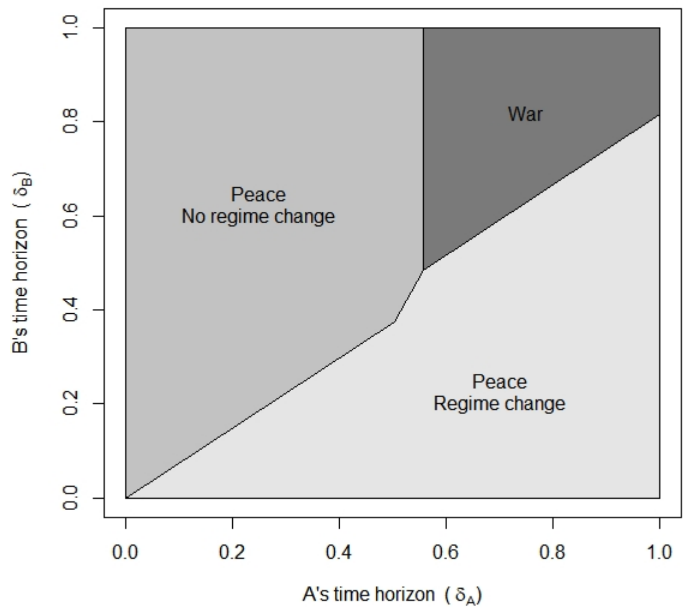
However, the war ranges are also considerably more complex, due to the asymmetric impact of regime change discussed above. There is a remnant of the war range in figure 1, which follows the previous logic. The main the main war range (on the center-left) is also more complex than the other models. When π_2 is low, the initial B regime is willing to give A a substantial portion of the disputed good in the present to prevent war and the possibility of regime change. If regime change occurs, A's additional benefits in stage 2 are only the difference between the B regimes resolve, c_{Bi} and c_{Br}. Thus A is willing to accept additional benefits in the present to avoid war. However, as π_2 increases, the initial B regime's benefits in the second stage would also decrease, making them more willing to risk regime change in war. A's increased benefits of regime change do not initially decrease, as they are only dependent on the difference between the B regimes' resolves. Thus, A's willingness to fight does not initially change while B's willingness to fight increases, making war more likely. Only when π_2 increases to such an extent that the replacement B regime would give A the entire disputed good in the second stage do A's benefits of regime change start to decrease, then making war less likely.

Figure 3:
Time Horizons and War, no power shift
Regime Change Model
$\pi_1 = \pi_2 = 0.2$, $\rho = 0.8$, $c_A = 0.01$, $c_{Bi} = 0.01$, $c_{Br} = 0.6$,



The relationship between time horizons and war in the regime change model, shown in figures 3 and 4, is also broadly similar to the decisive war model and the strategic object model. The main differences are that war can occur when the two actors have the same time horizon ($\delta_A = \delta_B$) when A faces an adverse power shift, and that war when there is no

Figure 4:
Time Horizons and War, A faces adverse power shift
Regime Change Model
$\pi_1 = 0.8$, $\pi_2 = 0.2$ $\rho = 0.8$, $c_A = 0.01$, $c_{Bi} = 0.01$, $c_{Br} = 0.6$



power shift requires a more significant difference in the actors time horizons. Both of these differences are due to the asymmetric effects of regime change, where the initial B regime would lose all benefits in the second stage while the A would only gain some benefits. Thus, when there is no power shift, B is more willing to trade a greater portion of the disputed good to avoid war and regime change, while A is more willing to make such an offer. Conversely, when A faces an adverse power shift, it is harder for A to induce the initial B regime to cede power peacefully as B would lose all benefits in the second stage. However, the adverse power shift A may still find war preferable to making the offer without regime change, even if the actors have the same time horizon.

B Full solution and proofs for the military capabilities model

In this appendix, I will provide the full solution and proofs for the military capabilities model.

In the second stage, A will make and B will accept the offers in lemma B.1, and in the first stage, A will make and B will accept the offer in lemma B.2.

Lemma B.1. *In the second stage of the military capabilities model, A will make the following offers. If there was a peaceful settlement in the first stage, A will offer* $x_{2,settled} = \pi_2 + m + c_B$ *or* $x_{2,settled} = 1$*, whichever is less. If A won a war in the first stage, A will offer* $x_{2,Awon} = \pi_2 + \mu + c_B$ *or* $x_{2,Awon} = 1$*, whichever is less. If B won a war in the first stage, A will offer* $x_{2,Bwon} = \pi_2 + c_B$ *or* $x_{2,Bwon} = 1$*, whichever is less. B will accept the relevant offer.*

Lemma B.2. *In the first stage of the military capabilities model, the* x_1 *of A's offer will be:* $x_1 = \pi_1 + \pi_1 \delta_B x_{2,Awon} + (1 - \pi_1)\delta_B x_{2,Bwon} - \delta_B x_{2,settled} + c_B$ *of* $x_1 = 1$*, whichever is less. If* $\delta_A \geq \delta_B$*, the* m *component of A's offer will be* $m = \frac{\pi_1 - \delta_B \pi_2 + \pi_1 \delta_B x_{2,Awon} + (1-\pi_1)\delta_B x_{2,Bwon} + (1-\delta_B)c_B}{\delta_B}$ *, or* $m = 1 - \pi_2 - c_B$*, whichever is smallest. If* $\delta_A \leq \delta_B$*, the* m *component of A's offer will be* $m = 0$ *or* $m = \frac{\pi_1 + \pi_1 \delta_A x_{2,Awon} + (1-\pi_1)\delta_A x_{2,Bwon} - c_A - \delta_A \pi_2 - \delta_A c_B - 1}{\delta_A}$*, whichever is largest. B will accept the relevant offer.*

The values for stage 2 in lemma B.1 are straightforward. B will accept any offer that gives them a greater expected value than war. Thus, B will accept any offer that meets the following conditions: $1 - x_{2,settled} \geq 1 - \pi_2 - m - c_B$ if a settlement was reached, $1 - x_{2,Awon} \geq 1 - \pi_2 - \mu - c_B$ if A won a war, and $1 - x_{2,Bwon} \geq 1 - \pi_2 - c_B$ if B won the war. These simplify to the values in lemma B.1. A will make the maximum offer that B will accept, provided it is preferable to war. A will be willing to make offers that meet the following conditions: $x_{2,settled} \leq \pi_2 + m - c_A$ if a settlement was reached, $x_{2,Awon} \leq \pi_2 + \mu - c_A$ if A won a war, and $x_{2,Bwon} \leq \pi_2 - c_A$ if B won the war. It is obvious that the offers in lemma B.1 meet these conditions.

In the stage 1, B will accept any value that gives them a greater expected value than war. Thus, B will accept and offer $1 - x_1 + \delta_B(1 - x_{2,settle}) \geq \pi_1(0 + \delta_B(1 - x_{2,Awin})) + (1 - \pi_1)(1 + \delta_B(1 - x_{2,Bwin})) - c_B$. Simplifying gives the value for x_1 in lemma B.2. A will be willing to make any offer that gives them a higher expected value than war. Therefore, A will be willing to make any offer that meets the following condition: $x_1 + \delta_A x_{2,settle} \geq \pi_1(1 + \delta_A x_{2,Awin}) + (1 - \pi_1)(0 + \delta_A x_{2,Bwin}) - c_A$. Inserting the values from lemma B.1, simplifying shows that A will prefer the offer in lemma B.2 to war for at least some value of m.

It is also necessary to determine which value of m A will choose. A's expected value for making the offer that B will barely accept is $x_1 + \delta_A x_{2,settled}$. Inserting the values for x_1 and $x_{2,settled}$ gives: $\pi_1 + \pi_1\delta_B x_{2,Awon} + (1 - \pi_1)\delta_B x_{2,Bwon} + c_B + (\delta_A - \delta_B)(\pi_2 + m + c_B)$. Note that this is linearly increasing with respect to m if $\delta_A > \delta_B$ and linearly decreasing if $\delta_A < \delta_B$. Therefore, A prefers the maximum value of m when $\delta_A > \delta_B$ and the minumum when $\delta_A < \delta_B$.

Setting the value of x_1 to 0 and solving for m gives the maximum value of m that B will accept: $m = \frac{\pi_1 + \pi_1\delta_B x_{2,Awon} + (1-\pi_1)\delta_B x_{2,Bwon} - \delta_B\pi_2 + c_B - \delta_B c_B}{\delta_B}$. A will also be unwilling to offer any value m greater than $1 - \pi_2 - c_B$, since if $\pi_2 + m + c_B > 1$, A can still only get 1 in stage 2. These are the values for in lemma B.2 for $\delta_A \geq \delta_B$. Setting the value of x_1 to 1 in A's condition for war and solving for m gives the minimum value of m that A will be willing to offer: $m = \frac{\pi_1(1+\delta_A x_{2,Awin}) + (1-\pi_1)(0+\delta_A x_{2,Bwin}) - c_A - \delta_A c_B - \delta_A\pi_2 - 1}{\delta_A}$. A will offer this or $m = 0$, whichever is largest. These are the values for in lemma B.2 for $\delta_A \leq \delta_B$.[1]

[1] If all the values in lemma B.1 are greater than 1, then the exact value of m in stage 1 is irrelevant.

C Full solution and proofs for the resource rich territory model

In this appendix, I will provide the full solution for the resource rich territory model, and proofs that it is correct.

In the second stage, A will make and B will accept the offers in lemma C.1. Similarly, in the first stage, A will make and B will accept the offer in lemma C.2.

Lemma C.1. *In the second stage of the resource rich territory model, A will make the following offers. If there was a peaceful settlement in the first stage, A will offer* $x_{2,settled} = \frac{f_{a,2}+m_2*x_1}{f_{a,2}+f_{b,2}+m_2} + c_B$ *or* $x_{2,settled} = 1$*, whichever is less. If A won a war in the first stage, A will offer* $x_{2,Awon} = \frac{f_{a,2}+m_2}{f_{a,2}+f_{b,2}+m_2} + c_B$ *or* $x_{2,Awon} = 1$*, whichever is less. If B won a war in the first stage, A will offer* $x_{2,Bwon} = \frac{f_{a,2}}{f_{a,2}+f_{b,2}+m_2} + c_B$ *or* $x_{2,Bwon} = 1$*, whichever is less. B will accept the relevant offer.*

Lemma C.2. *In stage 1 of the resource model, A will offer* $x_1 = \frac{f_{a,1}+m_1*q_1}{f_{a,1}+f_{b,1}+m_1}(1+\delta_B x_{2,Awon}) + (1 - \frac{f_{a,1}+m_1*q_1}{f_{a,1}+f_{b,1}+m_1})\delta_B x_{2,Bwon} - \delta_B x_{2,settled} + c_B$ *or* $x_1 = 1$*, whichever is less. B will accept the offer.*

The values in lemma C.1, describing the strategies in the 2nd stage, are straightforward. B will accept any offer that gives them a greater expected value than war. Thus, B will accept any offer that meets the following condition: $1 - x_2 = 1 - \pi_2 - c_B$, with π_2 representing A's probability of winning in the second stage. A's probability of winning is: $\pi_{2,settled} = \frac{f_{a,2}+m_2*x_1}{f_{a,2}+f_{b,2}+m_2}$ if a settlement was reached, $\pi_{2,Awon} = \frac{f_{a,2}+m_2}{f_{a,2}+f_{b,2}+m_2}$ if A won a war, and $\pi_{2,Bwon} = \frac{f_{a,2}}{f_{a,2}+f_{b,2}+m_2}$ if B won the war. Inserting these values into B's acceptance condition and simplifying gives the values in lemma C.1. A will make the maximum offer that B will accept, provided it is preferable to war. Thus, A will be willing to make offers that meet the following condition: $x_{2,settle} \leq \pi_2 - c_A$. After inserting A's probabilities of winning and simplifying shows that the offers in lemma C.1 meet these conditions.

In the first stage, B will likewise accept any offer that gives them a greater expected value than war. Thus, B will accept any offer that meets the following condition: $(1 - x_1) + \delta_B(1 - x_{2,settle}) \geq \pi_1(0 + \delta_B(1 - x_{2,Awins})) + (1 - \pi_1)(1 + \delta_B(1 - x_{2,Bwins})) - c_B$, with π_1 representing A's probability of winning in the first stage. A's probability of winning is $\pi_1 = \frac{f_{a,1}+m_1*q_1}{f_{a,1}+f_{b,1}+m_1}$. Substituting this gives the offers in lemma C.2. A will make this offer if it is higher than A's expected value for war. Thus, A will make the offer if $x_1 v_1 + \delta_A x_{2,settle} v_2 \geq \pi_1(1 * v_1 + \delta_A x_{2,Awins} v_2) + (1 - \pi_1)(0 * v_1 + \delta_A x_{2,Bwins} v_2) - c_A$. Inserting the offers in lemma C.1 and A's probability of winning from the previous paragraph shows that A always prefers the offers in C.2 to war.

D Full solution and proofs for the defensive advantage model

In this appendix, I will provide the full solution for the defensive advantage model, and proofs that it is correct.

In the second stage, A will make and B will accept the offers in lemma D.1. Similarly, in the first stage, A will make and B will accept the offer in lemma D.2.

Lemma D.1. *In the second stage of the defensive advantage model, A will make the following offers. If a settlement was reached, A will offer* $x_{2,settled} = \frac{\pi_2}{\alpha_2} + \frac{\alpha_2 - 1}{\alpha_2} x_1 + c_B$ *or* $x_2 = 1$*, whichever is less If A won a war, A will offer* $x_{2,Awon} = \frac{\pi_2}{\alpha_2} + \frac{\alpha_2 - 1}{\alpha_2} + c_B$ *or* $x_2 = 1$*, whichever is less If B won a war, A will offer* $x_{2,Bwon} = \frac{\pi_2}{\alpha_2} + c_B$ *or* $x_2 = 1$*, whichever is less If a war resulted in a stalemate, A will offer* $x_{2,stalemate} = \frac{\pi_2}{\alpha_2} + \frac{\alpha_2 - 1}{\alpha_2} q_1 + c_B$ *or* $x_2 = 1$*, whichever is less. B will accept the relevant offer*

Lemma D.2. *In the defensive advantage model, A will offer* $x_1 = \frac{\pi_1}{\alpha_1} + \frac{\alpha_1 - 1}{\alpha_1} q_1 - \delta_B x_{2,settled} + \frac{\pi_1}{\alpha_1} \delta_B x_{2,Awon} + \frac{1 - \pi_1}{\alpha_1} \delta_B x_{2,Bwon} + \frac{\alpha_1 - 1}{\alpha_1} \delta_B x_{2,stalemate} + c_B$ *or* $x_1 = 1$*, whichever is smaller. B will accept the offer.*

The 2nd stage offers in lemma D.1 are straightforward. B will accept any offer that gives them a greater expected value than war. Thus, B will accept any offer that meets the following condition: $1 - x_2 \geq \frac{\pi_2}{\alpha_2} * 0 + \frac{1-\pi_2}{\alpha_2} * 1 + \frac{\alpha_2 - 1}{\alpha_2}(1 - q_2) - c_B$. The status quo in the second stage, q_2 is x_1 if a settlement was reached in the first stage, 1 if A won a war, and 0 if B won a war. Inserting these values into B's acceptance condition and simplifying gives the values in lemma D.1. A will make the maximum offer B will accept, provided it is preferable to war. Thus, A will be willing to make offers that meet the following conditions: $x_2 \geq \frac{\pi_2}{\alpha_2} * 1 + \frac{1-\pi_2}{\alpha_2} * 0 + \frac{\alpha_2 - 1}{\alpha_2} * q_2 - c_A$. Inserting the second stage status quo and simplifying shows that the offers in lemma D.1 meet this condition.

In the first stage, B will accept any offer that gives them a greater expected value than war, so B will accept if: $1 - x_1 + \delta_B(1 - x_{2,settled}) \geq \frac{\pi_1}{\alpha_1}(0 + \delta_B(1 - x_{2,Awon})) + \frac{1-\pi_1}{\alpha_1}(1 + \delta_B(1 - x_{2,Bwon})) + \frac{\alpha_1 - 1}{\alpha_1}(1 - q_1 + \delta_B(1 - x_{2,stalemate})) - c_B$. Simplification gives the offers in lemma D.2. A will make this offer if it gives them a greater expected value than war, so A will make the offer if $x_1 + \delta_A x_{2,settled} \geq \frac{\pi_1}{\alpha_1}(1 + \delta_A x_{2,Awon}) + \frac{1-\pi_1}{\alpha_1}(0 + \delta_A x_{2,Bwon}) + \frac{\alpha_1 - 1}{\alpha_1}(q_1 + \delta_A x_{2,stalemate}) - c_A$. Inserting the values from D.1 and simplifying shows that the offer in D.2 is always mutually preferable to war.

E Full solution and proofs for the decisive war model

In this appendix, I will provide the full solution for the decisive war model, and proofs that it is correct.

In the second stage, A will make and B will accept the offers in lemma E.1. In the first stage, A will make an offer not demanding B surrender under the conditions detailed in lemma E.2. A will make an offer demanding B surrender under the conditions detailed in lemma E.3. A will make an unserious offer they know B will reject, resulting in war, under the conditions in lemma E.4.

Lemma E.1. *If a second stage occurs in the decisive war model, A will make the offer*

$x_2 = \pi_2 + c_B$ or $x_2 = 1$, whichever is less. B will accept the offer.

Lemma E.2. *In the first stage of the decisive war model, there may be an equilibrium where A refrains from demanding B's surrender and offers: $x_{1,no-surrender} = \pi_1 + \pi_1\kappa\delta_B - \pi_1\kappa\delta_B x_2 + c_B$ or $x_{1,no-surrender} = 1$, whichever is less. B accepts the offer.*

This equilibrium will exist if: $x_{1,no-surrender} + \delta_A x_2 \geq x_{1,surrender} + \delta_A$ and $x_{1,no-surrender} \geq \pi_1 + \pi_1\kappa\delta_A - \pi_1\kappa\delta_A x_2 - c_A$

Lemma E.3. *In the first stage of the decisive war model, there may be an equilibrium where A does demand B's surrender and offers: $x_{1,surrender} = \pi_1 - (1-\pi_1\kappa)\delta_B + (1-\pi_1\kappa)\delta_B x_2 + c_B$ or $x_{1,surrender} = 1$, whichever is less. B accepts the offer.*

This equilibrium will exist if: $x_{1,surrender} + \delta_A \geq x_{1,no-surrender} + \delta_A x_2$ and $x_{1,surrender} \geq \pi_1 - (1-\pi_1\kappa)\delta_A + (1-\pi_1\kappa)\delta_A x_2 - c_A$ and $x_{1,surrender} \geq 0$

Lemma E.4. *In the first stage of the decisive war model, A will make an unserious offer and B will reject if:*

$x_{1,no-surrender} \leq \pi_1 + \pi_1\kappa\delta_A - \pi_1\kappa\delta_A x_2 - c_A$ and $x_{1,surrender} < 0$

or

$x_{1,surrender} \leq \pi_1 - (1+\pi_1\kappa)\delta_A + (1-\pi_1\kappa)\delta_A x_2 - c_A$ and $1 \leq \pi_1 + \pi_1\kappa\delta_A - \pi_1\kappa\delta_A x_2 - c_A$,

The strategies in the second stage are straightforward. B is willing to make any offer that gives them a greater expected value than war. Therefore, B will accept any offer that meets the following condition: $1 - x_2 \geq 1 - \pi_2 - c_B$, which simplifies to the offer in lemma E.1. A is willing to make this offer as long as it gives a higher expected value than war. Thus, A will make the offer if $x_2 \geq \pi_2 - c_A$. The offer in lemma E.1 always meets this condition.

In the first stage, B is willing to accept any offer that gives them a higher expected value than war. Thus, B is willing to accept an offer where they do not surrender that meets the following condition: $1 - x_{1,no-surrender} + \delta_B(1-x_2) \geq \pi_1(0 + \kappa * 0 + (1-\kappa)\delta_B(1-x_2)) + (1-\pi_1)(1+\delta_B(1-x_2)) - c_B$. This simplifies to the offer in lemma E.2. Replacing

$x_{1,no-surrender} = 0$ in the above inequality shows that there is always some offer where B does not surrender that B is willing to accept.

Likewise, B is willing to accept an offer where they surrender that meets the following condition: $1 - x_{1,surrender} + \delta_B * 0 \geq \pi_1(0 + \kappa * 0 + (1-\kappa)\delta_B(1-x_2)) + (1-\pi_1)(1+\delta_B(1-x_2)) - c_B$. This simplifies to the offer in lemma E.3. Replacing $x_{1,no-surrender} = 0$ in the above inequality and simplifying shows that there may be no offer that B is willing to accept that demands B's surrender. The second condition in lemma E.3 is the requirement that there is some offer demanding B's surrender that they will prefer to war.

Given the offers that B is willing to accept, A potentially has three options: make an offer that B will accept where B does not surrender, make an offer that B will accept where B does surrender (assuming one exists), or make an unserious offer that B will reject and causing war. A will make the offer that gives them the greatest expected value.

Thus, A will make the offer not demanding B's surrender in lemma E.2 if it is overall better than the offer demanding B's surrender and better than war. The offer will be better for A than the offer demanding B's surrender if $x_{1,no-surrender} + \delta_A x_2 \geq x_{1,surrender} + \delta_A * 1$. This is the first condition in lemma E.2. The offer will be better than war if $x_{1,no-surrender} + \delta_A x_2 \geq \pi_1(1 + \kappa * \delta_A * 1 + (1-\kappa)\delta_A x_2) + (1-\pi_1)(0 + \delta_A x_2) - c_A$. This simplifies to the second condition in lemma E.2.

Similarly, A will make the offer demanding B's surrender in lemma E.3 if it is better than both the offer not demanding B's surrender and better than war. The offer will be better for A than the offer not demanding B's surrender if $x_{1,surrender} + \delta_A * 1 \geq x_{1,no-surrender} + \delta_A x_2$. This simplifies to the first condition in lemma E.3. The offer will be better than war if $x_{1,surrender} + \delta_A * 1 \geq \pi_1(1 + \kappa * \delta_A * 1 + (1-\kappa)\delta_A x_2) + (1-\pi_1)(0 + \delta_A x_2) - c_A$. This simplifies to the second condition in lemma E.3. In addition, as noted above, there may be no offer demanding B's surrender that B will accept. This is the final condition in lemma E.3.

A will make an unserious offer that B rejects resulting in war if A prefers war to any available offer. However, simple calculations show that if $x_{1,surrender} \geq 0$, A always prefers this offer to war if $\delta_A \geq \delta_B$. Similarly, simple calculations show that if $x_{1,no-surrender} \leq 1$ A always prefers this offer to war if $\delta_A \geq \delta_B$. Thus, war only occurs if $x_{1,surrender} < 0$ or $x_{1,no-surrender} > 1$. In addition, it is impossible for $x_{1,surrender} < 0$ and $x_{1,no-surrender} > 1$ to be simultaneously true. Calculating the maximum value of π_1 that would make $x_{1,surrender} < 0$ true, and inserting this into the formula for $x_{1,no-surrender}$ shows that the latter would always be less than 1.

Thus, war can occur only in two conditions. First, if $x_{1,surrender} < 0$ such that B will not accept any offer where B surrenders and $x_{1,no-surrender} + \delta_A x_2 < \pi_1(1 + \kappa * \delta_A * 1 + (1 - \kappa)\delta_A x_2) + (1 - \pi_1)(0 + \delta_A x_2) - c_A$ such that A prefers war to the offer where B does not surrender. This pair of inequalities is the first case of war in lemma E.4. War could also occur if $x_{1,no-sur} > 1$, A prefers war to the offer with no-surrender ($x_{1,no-surrender} + \delta_A * x_2 < \pi_1(1 + \kappa * \delta_A * 1 + (1 - \kappa)\delta_A x_2) + (1 - \pi_1)(0 + \delta_A x_2) - c_A$, with $x_{1,no-surrender} = 1$), and A prefers war to the offer with surrender($x_{1,surrender} + \delta_A * 1 < \pi_1(1 + \kappa * \delta_A * 1 + (1 - \kappa)\delta_A x_2) + (1 - \pi_1)(0 + \delta_A x_2) - c_A$). This is the second set of inequalities in lemma E.4. Inserting the values for $x_{1,no-surrender}$, $x_{1,surrender}$, and x_2 and simplifying gives the conditions for war in proposition 4 in the main paper.

F Full solution and proofs for the regime change model

In this appendix, I will provide the full solution for the regime change model, and proofs that it is correct.

In the second stage, A will make and B will accept the offers in lemma F.1. In the first stage, A will make an offer not demanding B step down under the conditions detailed in lemma F.2. A will make an offer demanding B step down under the conditions detailed in lemma F.3. A will make an unserious offer they know B will reject, resulting in war, under the conditions in lemma F.4.

Lemma F.1. *In the second stage of the regime change model, If the initial regime of B* *remains in power, A will offer* $x_{2i} \leq \pi_2 + c_{Bi}$ *or* $x_{2i} = 1$*, whichever is less. The initial B regime will accept the offer. If regime change occured in the first round, such that the replacement regime of B plays in the second stage, A will make offer* $x_{2r} \leq \pi_2 + c_{Br}$ *or* $x_{2r} = 1$*, whichever is less. The replacement B regime will accept the offer.*

Lemma F.2. *In the first stage of the regime change model, there may be an equilibrium where A does not demanding regime change and offers:* $x_{1,nochange} = \pi_1 + \pi_1\rho\delta_{Bi}(1 - x_{2i}) + c_{Bi}$ *or* $x_{1,nochange} = 1$*, whichever is less. The initial B regime accepts the offer.*

This equilibrium will occur under the following conditions: $x_{1,nochange} + \delta_A x_{2i} \geq x_{1,regimechange} + \delta_A x_{2r}$ *and* $x_{1,nochange} \geq \pi_1 + \pi_1\rho\delta_A x_{2r} - \pi_1\rho\delta_A x_{2i} - c_A$

Lemma F.3. *In the first stage of the regime change model, there may be an equilibrium where A does demand regime change and offers:* $x_{1,regimechange} = \pi_1 - \delta_{Bi}(1 - \pi_1\rho)(1 - x_2 i) + c_{Bi}$ *or* $x_{1,regimechange} = 1$*, whichever is less. The initial B regime accepts the offer.*

This equilibrium will exist under the following conditions: $x_{1,regimechange} + \delta_A x_{2r} \geq x_{1,nochange} + \delta_A x_{2i}$ *and* $x_{1,regimechange} \geq \pi_1 - (1 - \pi_1\rho)\delta_A x_{2r} + (1 - \pi_1\rho)\delta_A x_{2i} - c_A$ *and* $x_{1,regimechange} \geq 0$

Lemma F.4. *In the first stage of the regime change model, A will make an unserious offer and the initial B regime will reject if:*

$x_{1,nochange} < \pi_1 + \pi_1\rho\delta_A x_{2r} - \pi_1\rho\delta_A x_{2i} - c_A$ *and* $x_{1,regimechange} < 0$

or

$x_{1,regimechange} < \pi_1 - (1 - \pi_1\rho)\delta_A x_{2r} + (1 - \pi_1\rho)\delta_A x_{2i} - c_A$ *and* $1 < \pi_1 + \pi_1\rho\delta_A x_{2r} - \pi_1\rho\delta_A x_{2i} - c_A$

The strategies in the second stage are straightforward. Both regime types of B are willing to make any offer that gives them a greater expected value than war. Therefore, the initial B regime will accept any offer that meets the following condition: $1 - x_2 \geq 1 - \pi_2 - c_{Bi}$, and the replacement regime of B will accept any offer that meets the following condition: $1 - x_2 \geq 1 - \pi_2 - c_{Br}$. These simplify to the offers in lemma F.1. A is willing to make this offer as long as it gives a higher expected value than war. Thus, A will make the offers if $x_2 \geq \pi_2 - c_A$. The offers in lemma F.1 always meets this condition.

In the first stage, the initial B regime is willing to accept any offer that gives them a higher expected value than war. Thus, they are willing to accept an offer where they remain in power that meets the following condition: $1 - x_{1,nochange} + \delta_{Bi}(1 - x_2 i) \geq \pi_1(0 + \delta_{Bi}\rho * 0 + \delta_{Bi}(1-\rho)(1-x_{2i})) + (1-\pi_1)(1+\delta_{Bi}(1-x_2 i)) - c_{Bi}$ This simplifies to the offer in lemma F.2. Replacing $x_{1,nochange} = 0$ in the above inequality shows that there is always some offer where B does not surrender that B is willing to accept.

Likewise, the initial B regime is willing to accept an offer where they step down and regime change occurs that meets the following condition: $1 - x_{1,regimechange} + \delta_{Bi} * 0 \geq \pi_1(0 + \delta_{Bi}\rho * 0 + \delta_{Bi}(1-\rho)(1-x_{2i})) + (1-\pi_1)(1+\delta_{Bi}(1-x_2 i)) - c_{Bi}$ This simplifies to the offer in lemma F.3. Replacing $x_{1,regimechange} = 0$ in the above inequality and simplifying shows that there may be no offer that B is willing to accept that demands regime change. This is the final condition for the equilibrium where B peacefully surrenders in lemma F.3.

Given the offers that B will accept, A has three options in the first stage. A can make the offer without demanding B step down, they can make the offer demanding regime change, or they can make an unserious offer that B will reject, resulting in war. A will make the offer that gives them the greatest overall expected utility.

Thus, A will make the offer in lemma F.2 not demanding regime change if it is better than both the offer demanding regime change and war. The offer is better than the offer demanding regime change if $x_{1,remain} + \delta_A x_{2r} \geq x_{1,regimechange} + \delta_A x_{2i}$, which is the first condition in lemma F.2. The offer is better than war if $x_1 + \delta_A x_{2i} \geq \pi_1(1 + \delta_A \rho x_{2r} + \delta_A(1-\rho)x_{2i}) + (1-\pi_1)(0 + \delta_A x_{2i}) - c_A$, which simplifies to the second condition in lemma F.2.

Likewise, A will make the in lemma F.3 demanding regime change (assuming one is possible) if it is both better than the offer demanding regime change and better than war. The offer is better than the offer not demanding regime change if $x_{1,nochange} + \delta_A x_{2r} \geq x_{1,regimechange} + \delta_A x_{2i}$, which is the first condition in lemma F.3. The offer is better than war if $x_{1,nochange} + \delta_A x_{2r} \geq \pi_1(1 + \delta_A \rho x_{2r} + \delta_A(1-\rho)x_{2i}) + (1-\pi_1)(0 + \delta_A x_{2i}) - c_A$ which simplifies to the second condition in lemma F.3. Finally, as noted above, there may be no offer demanding regime change that the initial regime of B will accept. The final condition in lemma F.3 is the condition under which there is some offer demanding regime change that B will accept.

War will occur if it A finds it preferable to to both the offer demanding regime change and the offer not demanding regime change. If $x_{1,nochange} \leq 1$, A always prefers this offer to war if $\delta_A(x_{2r} - x_{2i}) \geq \delta_{Bi}(1 - x_{2i})$. If $x_{1,remain} \geq 0$, A always prefers this offer to war if $\delta_A(x_{2r} - x_{2i}) \leq \delta_{Bi}(1 - x_{2i})$ Therefore, war will only occur if either $x_{1,regimechange} \geq 1$ or $x_{1,remain} \leq 0$. In addition, it is impossible for both $x_{1,nochange} \geq 1$ and $x_{1,regimechange} \leq 0$ to be true at the same time. Simple calculations show that the maximum value for $x_{1,regimechange} \leq 0$ is $\frac{\delta_{Bi}(1-x_2 i)-c_{Bi}}{(1+\rho\delta_{Bi}(1-x_2 i))}$. Inserting this value into the formula for $x_{1,nochange}$ (which increases as π_1 increases) and simplifying gives $\delta_{Bi}(1-x_2 i)$, and so when $x_{1,regimechange} \leq 0$, it is always true that $x_{1,nochange} \leq 1$.

Thus war can happen in two cases. First if there is no offer demanding regime change that B will accept ($x_{1,regimechange} < 0$) and A prefers war to the offer not demanding regime change. This is the first set of inequalities in lemma F.4. Second, war can occur if $x_{1,regimechange} > 1$, A prefers war to offering 1 while allowing the initial regime to remain in place, and A prefers war to the acceptable offer demanding regime change. This is the second set of inequalities in lemma F.4. Inserting the values for $x_{1,nochange}$, $x_{1,regimechange}$, x_{2i} and x_{2i} and simplifying gives the war conditions in proposition 5 in the main paper.

G Full solution and proofs for the strategic object model

In this appendix, I will provide the full solution for the indivisible strategic object model, and proofs that it is correct.

In the second stage, A will make and B will accept the offers in lemma G.1. In the first stage, A will make an offer not demanding the strategic objecte under the conditions detailed in lemma G.2. A will make an offer demanding demanding the strategic object under the conditions detailed in lemma G.3. A will make an unserious offer they know B will reject, resulting in war, under the conditions in lemma G.4.

Lemma G.1. *In the second stage of the strategic object model where A gained the strategic object in the first stage, A will offer* $x_{2,Agets} = \pi_2 + \sigma + c_B$ *or* $x_{2,Agets} = 1$*, whichever is less. If B gained the strategic object, A will offer:* $x_{2,Bgets} = \pi_2 + c_B$ *or* $x_{2,Bgets} = 1$*, whichever is less. B will accept the relevant offer.*

Lemma G.2. *In the first stage of the strategic object model, there may be an equilibrium where A allows B to have the strategic object, with A offering:* $x_{1,Bgets} = \pi_1 + \pi_1\delta_B(x_{2,Agets} - x_{2,Bgets}) + c_B$ *or* $x_{1,Bgets} = 1$*, whichever is less. B accepts the offer.*

This equilibrium will occur under the following conditions: $x_{1,Bgets} + \delta_A x_{2,Bgets} \geq x_{1,Agets} + \delta_A x_{2,Agets}$ *and* $x_{1,Bgets} \geq \pi_1 + \pi_1\delta_A(x_{2,Agets} - x_{2,Bgets}) - c_A$

Lemma G.3. *In the first stage of the strategic object model, there may be an equilibrium where A demands the strategic object:* $x_{1,Agets} = \pi_1 - \delta_B(1 - \pi_1)(x_{2,Agets} - x_{2,Bgets}) + c_B$ *or* $x_{1,Agets} = 1$*, whichever is less. B accepts the offer.*

This equilibrium will exist under the following conditions: $x_{1,Agets} + \delta_A x_{2,Agets} \geq x_{1,Bgets} + \delta_A x_{2,Bgets}$ *and* $x_{1,Agets} \geq \pi_1 - \delta_A(1 - \pi_1)(x_{2,Agets} - x_{2,Bgets}) - c_A$ *and* $x_{1,Agets} \geq 0$

Lemma G.4. *In the first stage of the strategic object model, A will make an unserious offer, B will reject, and war will occur if:*

$x_{1,Bgets} \leq \pi_1 + \pi_1\delta_A(x_{2,Agets} - x_{2,Bgets}) - c_A$ *and* $x_{1,Agets} < 0$

or

$x_{1,Agets} \leq \pi_1 - \delta_A(1 - \pi_1)(x_{2,Agets} - x_{2,Bgets}) - c_A$ *and* $1 \leq \pi_1 + \pi_1\delta_A(x_{2,Agets} - x_{2,Bgets}) - c_A$

The strategies in the second stage are straightforward. B is willing to make any offer that gives them a greater expected value than war. If A received the strategic object after the first stage, B will accept any offer that meets the following condition: $1 - x_{2,Agets} \geq$

$1-\pi_2-\sigma-c_B$. If B received the strategic object after the first stage, B will accept any offer that meets the following condition: $1-x_{2,Bgets} \geq 1-\pi_2-c_B$. These simplify to the offer in lemma G.1. A is willing to make this offer as long as it gives a higher expected value than war. Thus, A will make the offer if $x_{2,Agets} \geq \pi_2+\sigma-c_A$ if A received the strategic object, and $x_{2,Bgets} \geq \pi_2-c_A$ if B received the strategic object. It is obvious that the offers in lemma G.1 always meets these conditions.

In the first stage, B is willing to accept any offer that gives them a higher expected value than war. Thus, B is willing to accept an offer where they retain the strategic object that meets the following condition: $1-x_{1,Bgets}+\delta_B(1-x_{2,Bgets}) \geq \pi_1(0+\delta_B(1-x_{2,Agets}))+(1-\pi_1)(1+\delta_B(1-x_{2,Bgets}))-c_B$. This simplifies to the offer in lemma G.2. Replacing $x_{1,Bgets}=0$ in the above inequality shows that there is always some offer with B retaining the strategic object that B is willing to accept.

Likewise, B is willing to accept an offer where A gets the strategic object that meets the following condition: $1-x_{1,Agets}+\delta_B(1-x_{2,Agets}) \geq \pi_1(0+\delta_B(1-x_{2,Agets}))+(1-\pi_1)(1+\delta_B(1-x_{2,Bgets}))-c_B$. This simplifies to the offer in lemma G.3. Replacing $x_{1,Agets}=0$ in the above inequality and simplifying shows that there may be no offer that B is willing to accept that demands B's surrender. This is the final condition for the equilibrium where A gets the strategic object in lemma G.3.

Given the offers that B is willing to accept, A potentially has three options: make an offer that B will accept where B gets the strategic object, make an offer that B will accept where A gets the strategic object (assuming one exists), or make an unserious offer that B will reject and resulting in war. A will make the offer that gives them the greatest expected value.

Thus, A will make the offer where B gets the strategic object in lemma G.2 if it is overall better than the offer where A gets the strategic object and better than war. The offer will be better for A than the offer where A gets the strategic object if $x_{1,Bgets}+\delta_A x_{2,Bgets} \geq x_{1,Agets}+\delta_A x_{2,Agets}$. This is the first condition in lemma G.2. The offer will be better than war if $x_{1,Bgets} + \delta_A x_{2,Bgets} \geq \pi_1(1 + \delta_A x_{2,Agets}) + (1 - \pi_1)(0 + \delta_A x_{2,Bgets}) - c_A$. This simplifies to the second condition in lemma G.2.

Similarly, A will make the offer where A gets the strategic object in lemma G.3 if it is both better than the offer where B gets the strategic object and better than war. The offer will be better for A than the offer where A gets the strategic object if $x_{1,Agets} + \delta_A x_{2,Agets} \geq x_{1,Bgets} + \delta_A * x_{2,Bgets}$. This is the first condition in lemma G.3. The offer will be better than war if $x_{1,Agets} + \delta_A x_{2,Agets} \geq \pi_1(1 + \delta_A x_{2,Agets}) + (1 - \pi_1)(0 + \delta_A x_{2,Bgets}) - c_A$. This simplifies to the second condition in lemma G.3. In addition, as noted above, there may be no offer demanding B's surrender that B will accept. This is the final condition in lemma G.3.

A will make an unserious offer that B rejects resulting in war if A prefers war to any offer that B would accept. However, simple calculations show that if $x_{1,Agets} \geq 0$, A always prefers this offer to war if $\delta_A \geq \delta_B$. Similarly, simple calculations show that if $x_{1,Bgets} \leq 1$ A always prefers this offer to war if $\delta_A \leq \delta_B$. Thus, war only occurs if $x_{1,Agets} < 0$ or $x_{1,Bgets} > 1$. In addition, it is impossible for both $x_{1,Agets} < 0$ or $x_{1,Bgets} > 1$ to be simultaneously true. Calculating the maximum value of π_1 that would make $x_{1,Agets} < 0$ true, and inserting this into the formula for $x_{1,Bgets}$ shows that the latter would always be less than 1.

Thus, war can occur only in two conditions. First, if $x_{1,Agets} < 0$ such that B will not accept any offer where B surrenders and $x_{1,Agets} + \delta_A x_{2,Agets} < \pi_1(1 + \delta_A x_{2,Agets}) + (1 - \pi_1)(0 + \delta_A x_{2,Bgets}) - c_A$ such that A prefers war to the offer where B does not surrender. This pair of inequalities is the first case of war in lemma G.4. War could also occur if $x_{1,Bgets} > 1$, A prefers war to the offer with no-surrender, and A prefers war to the offer with surrender. This is the second set of inequalities in lemma G.4. Inserting the values for $x_{1,Agets}$, $x_{1,Bgets}$, $x_{2,Agets}$ and $x_{2,Bgets}$ and simplifying gives the war conditions in proposition 6 in the main paper.